THE ZUTPHEN QUADRANT
A Very Early Equal-Hour Instrument Excavated in The Netherlands

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Zutphen is not a name which is widely known outside the Netherlands. However, it is a small and vibrant city with a long history and it has been carefully excavated over many years. The discovery of a small copper-alloy quadrant during excavations in March/April 2013 (Fig. 1) has great significance because it is clearly delineated to show equal hours and yet can be dated by archaeological methods to c.1300, virtually a century before the previously recorded quadrants of this type, made for the circle of Richard II of England.

Zutphen History
There is evidence\(^1\)\(^2\) that Zutphen (52° 08’ N; 6° 12’ E) existed in the Roman period. By the Middle Ages, it was an important walled city on the River IJssel and a member of the Hanseatic League of trading cities: its layout is shown in the slightly later map by Blaeu in Fig. 2. It withstood several sieges, including the Battle of Zutphen in 1586 when Sir Philip Sidney was mortally wounded. It has many fine buildings and its medieval library, with chained books, is one of only five which still remain in the world.

The ditches and earthworks which surrounded the walled city were built in the late 9th century and enlarged in the 11th century. They were then gradually levelled to provide more space for expansion, initially in the mid 13th century using sand from the rampart. A thick dark humus-rich layer, with many datable objects such as coins, pots, etc. in it, then built up until, in the period 1300 to 1320, another layer of sand was applied so that the area could be used as a market with a later build-up of material capping the area and providing the modern surface. The present city is now several times the original size though the remaining Gothic buildings in the centre and the surrounding outskirts are carefully preserved within the modern plan.

The Excavation
Early in 2013, plans were made to provide two areas for planting trees in the city, in locations which were over the filled-in ditches (Fig. 3). Many previous archaeological excavations in the city had already shown that a wealth of important finds were likely to be exposed by the digging so archaeologists were on hand to monitor the work. The stratification of the layers had also already been established with the sequence of filling the original ditches dated close-

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**Fig. 1.** The Zutphen quadrant. Photo courtesy of B. Fermin.\(^1\)

**Fig. 2.** Map of Zutphen by Willem and Joan Blaeu, 1649. North is to the left.

**Fig. 3.** General view of one of two planting holes for trees within the old city of Zutphen. Photo from ref. 1.
ly by the many finds. Material from the excavation, lifted in layers, was examined by metal detector as it was removed. Many items were found during the work and the star find was the small quadrant which is described here. It was quickly identified as an instrument for indicating equal-hours and so the very early dating for such a device was highly significant. The stratification of the layers can be seen in Fig. 4. They date the quadrant securely to the period of 1300–1320. No other items of a similar ‘scientific’ nature have been found in the archaeological investigations of Zutphen.

**Description of the Quadrant**

The quadrant shown in Fig. 1 is evidently made of a copper-alloy and is engraved on one side only. It is unsigned and undated. It is 62 mm in radius and 1 mm thick. Although quite a simple device it has been well-made. Rather surprisingly, as shown in Fig. 5, both of the sights are still firmly in place, as is the swivel mount (still moveable) for the plumb-line. Only the plumb-line, bob and bead itself are missing.

The sights, with tapered holes of approximately 1 mm diameter, are expertly set into the quadrant and riveted in place: they have a decidedly ‘medieval’ appearance similar to those on some other early instruments. In particular, some of the quadrans novus likely to be contemporary with the Zutphen quadrant also show the same general shape with a long ‘wing’ extending down the back of the quadrant. In contrast, all of the Richard II equal-hour quadrants have much smaller sights on the surface of one side.

A schematic representation of the quadrant is in Fig. 6.

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Fig. 4. Stratification of layers – photograph and schematic. The quadrant was found in layer LP2.

Fig. 5. Photographs of the back of the quadrant, the sights, and the swivel fixing for the plumb-bob.

Fig. 6. Schematic diagram of the quadrant, courtesy of Elisa de Vries, Davy Kastelein and Michel Groothedde.

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(Guelders), dated to 1324 and 1326. Of course, this does not date the quadrant absolutely but it is significant that the local lettering style later in the century is significantly different.

The final set of lines are for indicating the time from the sun’s altitude and it is immediately clear that these are for equal hours (rather than the seasonal or temporal or unequal hours) because only the midday semicircle passes through the origin of the quadrant – all the other hour-arcs intersect the left-hand radius which indicates zero altitude. The centre from which the all-important midday semicircle has been drawn is clearly visible as a small depression on the right-hand radius.

There are no numerals on the quadrant – the scale for altitude and the lines for hours and for declination are unnumbered, though there are spaces for the altitudes. There are, though, several sets of strange ‘crescent’ engravings, some in positions where numbers might be expected: these are unexplained. Close inspection does not indicate that they are attempts to erase other engravings.

Background to Quadrants and Quadrant Types in Medieval Europe

The earliest quadrants in Europe were of the type known (afterwards) as the quadrans vetus (old quadrant). This design dates back to the 9th century in Baghdad but was described in Europe by Sacrobosco in the 12th century. The basic design is for unequal hours and it is, approximately, a universal design although it becomes slightly inaccurate as the latitude increases from the typical values found in Islamic regions to those of northern Europe. The design is characterised by a semicircular midday (noon) line and then a set of circular hour arcs each passing through the origin of the quadrant and with a centre lying on the right-hand radius. They cut the protractor scale at 15° steps (i.e. at 75° for the arc indicating 1 and 11 hours). They were thus very easy to draw. To use the device, the bead on the plumb-line had to be positioned to suit the current date (‘rectified’) and this required access to a table of the Sun’s noon altitudes.

A development of this basic design of quadrans vetus at the end of the 13th century, usually attributed to Profatius, uses a sliding cursor calibrated for the Sun’s declinations and which can be positioned to suit the current latitude, thus removing the need for sets of tables.

The move from unequal hours to equal hours by the general population seems to have occurred gradually throughout the 14th century, linked to the spread of tower clocks. Naturally, there was a need for a quadrant to indicate these hours and the obvious and simplest development was for a design based on the quadrans vetus. The lines on the resulting device were now latitude-dependent and thus it was usable only at a single location. It might reasonably be presumed that the declination scale centred on the origin of the quadrant would still be calculated on the same basis as that for the unequal-hour quadrant. Noon was still indicated as part of a semi-circular arc but the other hour-lines no longer passed through the origin of the quadrant. A table of noon altitudes or a date (declination) scale was needed to rectify the bead before use.

A possible example of an early hybrid quadrant showing two sets of lines, for unequal and equal hours (the former reversed), is an unfinished device in the Oxford Museum of the History of Science. This instrument is rather dismissively described as having a “certain charm as an historic artefact from the medieval period” in the catalogue entry but it deserves more detailed study.

Evidence for the existence of equal-hours quadrants early in the 14th century is given by a drawing at the bottom of a page of an illuminated manuscript now in the British Library (MS Burney 275 f. 390v) and shown in Fig. 7. Attributed to the ‘Meliacian Master’, it was painted in central France between 1309 and 1316. It shows various animals using a cylinder dial, an armillary sphere, an astrolabe and a quadrant. The instruments are quite realistically drawn and on the quadrant it can be seen that the hour-lines do not all pass through the origin but that some of them intersect the left-hand edge of the instrument, the chief characteristic of equal hours.

Up to now, the earliest extant equal-hour quadrants were a group of four instruments believed to have been made for King Richard II and dated (or datable) from 1396 to 1400. Fig. 8 shows the example currently in Dorchester, dated 1398. These are very high-quality instruments believed to have been made as gifts from the King to his favoured courtiers and thus not likely to have been used seriously for time-telling. The majority of the extant devices do not have a complete set of declination arcs for the date, just limiting curves for the solstices and a central equinox curve. Instead, they feature detailed tables of the Sun’s midday altitude for the local latitude engraved on the reverse side. The lack of other examples of this design is puzzling.
A totally different form of equal-hour quadrant was described, also by Profatius, at the the end of the 13th century – the quadrans novus.\textsuperscript{13} This was based on a folded astrolabe and was thus a rather sophisticated astronomical instrument. It was, though, rather difficult to use for simple time-telling and thus only a very small number of examples exists – one was unearthed in Canterbury in 2005.\textsuperscript{14}

There is a small amount of evidence of other types of medieval quadrant showing equal hours. One English example is the ‘Norfolk quadrant’ which, although not expertly made, shows both unequal and equal hours, the latter in a form akin to the much later ‘Gunter’s quadrant’ design.\textsuperscript{15}

**Geometry of the Zutphen Quadrant**

It is worth considering the method by which the quadrant would have been drawn and the information which was required to do it, using only Euclidian geometry with a straight edge, a pair of compasses and a scribbling point. An example of laying out just such an equal-hour quadrant is given in the manuscript Bodleian MS Ashmole 19 (f. 54).\textsuperscript{16}

The first task would be to construct the protractor scale. Once the bounding curve had been drawn parallel to the limb with the compasses, they would then be used, with the same opening, to strike arcs on the bounding curve from the crossing points with both the left- and right-hand radii. This would give the 30° and 60° points and, where the two arcs crossed, the 45° point bisecting the full quadrant. There is no geometric construction which produces an exact trisection of an angle but it is not difficult, with a little practice and trial-and-error, to achieve this to a good approximation. Having thus produced the 10° divisions, they could be bisected before finally dividing by eye down to individual degrees. It is notable that the maker of the Zutphen quadrant has not bothered to divide the scale fully, only the part needed for possible solar altitudes at his latitude, up to 63°.

The next step is to bisect the right-hand radius to find the centre for drawing the midday semicircle. Up to this point the construction is universal and needs no input data. To draw the declination lines there are several options. One method is to consult tables to know the noon altitude of the sun at the intended location at the entry of the Sun into each of the zodiac signs, the most important being for Aries and Libra, on the equinoxes. These altitudes are located on the protractor scale and then the point where that appropriate radius line intersects the midday semicircle determines the radius of the equinox line. It is then easy to engrave a set of arcs centred on the origin and passing through the appropriate intersection points.

Finally, the hour lines have to be drawn and again tables are needed to supply the Sun’s altitude on three dates – the equinoxes, the summer solstice and either the winter solstice or the last date when the sun is above the horizon at the specified hour. The maker would now assume that the hour line was a segment of a circle passing through these three points. This is not quite mathematically exact particularly at high latitudes\textsuperscript{17} but the errors are small and the simplification was universally made by instrument makers over several centuries, until computing technology was available. Euclid tells us that there is only one circle passing through a set of three points: finding its centre with compasses and straight-edge is an interesting exercise. Once found and the arc drawn, the quadrant is essentially complete.

Tables of the Sun’s altitude as a function of date (declination) for each equal hour which would be needed to draw the equal-hour version of a quadrans vetus are known in the literature, though not common. One English example dating from around 1400 was by Robert Stikford at the Abbey of St Albans.\textsuperscript{18} The table is for a single latitude. Clearly, on the evidence of the quadrants, there must have been other tables in existence a century before this.

Knowing the above sequence, it is possible to reverse-engineer the Zutphen quadrant to find a number of features, particularly the intended latitude, the value of the obliquity assumed by the supplier of the altitude tables, and the geometrical precision of the maker.

The first point of interest is the design latitude $\phi$ and this can be obtained by several methods. The simplest, most direct and possibly most accurate is to note that at the equinoxes, the Sun’s midday altitude $\alpha_{m}$ is simply\textsuperscript{19}

$$\alpha_{m} = 90^\circ - \phi$$

By drawing a line on an ‘accurate’\textsuperscript{20} photograph of the quadrant from the origin through the point where the declination line for the equinoxes crosses the noon semicircle and extending it to the limb, the value of $\alpha_{m}$ can be read off as 38.0°, implying a design latitude of 52.0°. This may be compared to the Zutphen latitude of 52.1°. There is some tolerance in the measured value because of the difficulties in defining exactly where the origin is located (it is hidden by the swivel) and the crossing of the equinox and noon lines. It is inconclusive whether the quadrant was actually designed for the city but it was certainly somewhere in northern Europe and it would certainly have been usable in Zutphen.

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Fig. 8. The 1398 Richard II quadrant in Dorchester. Photo by the author with acknowledgements to the Dorset County Museum.
A second method of extracting the design latitude is by noting that
\[ \varphi = 90° - \frac{1}{2}(\alpha_{ns} + \alpha_{nw}) \]
where \( \alpha_{ns} \) (or \( \alpha_{nw} \)) is the Sun's meridian altitude at the summer (winter) solstice. The values of 62.8° and 14.0° measured on the quadrant give a design latitude of 51.6°, which is similar to the previous value but indicates the tolerances of both the engraving and measuring processes.

The same values of \( \alpha_{ns} \) and \( \alpha_{nw} \) are also related by
\[ \varepsilon = \frac{1}{2}(\alpha_{ns} - \alpha_{nw}) \]
where \( \varepsilon \) is the value of the obliquity of the ecliptic implied by the layout. The value obtained here is 24.4°, which is rather larger than the values those of 24° or 23½° which are often found in medieval manuscripts. Again, the tolerances of both the measurements and the original layout need to be borne in mind.

The fact that the protractor scale is only fully divided up to an altitude of 63° also tends to confirm that the quadrant was delineated for use at a latitude of around 51 or 52° N. It also suggests that it was intended only as an altitude sundial and not, for example, for measuring the heights of towers or taking different astronomical measurements.

Turning now to the declination (date) lines, the method by which they were drawn may be assessed. The radii of the key arcs, for the equinoxes and solstices, were measured from circles fitted to a photograph imported into a CAD program (TurboCad™) and then normalised by dividing by the diameter of the noon semicircle. The values are shown in Table 1 which also shows the values from two of the Richard II quadrants (see Fig. 8 for an example). Also shown are the theoretical values calculated assuming an optimised latitude of 52° and an obliquity (equal to the declination on the solstices) of 24°.

The theoretical values are
\[ r_n = \sin(90° - \varphi + \delta) \]
where \( r_n \) is the normalised radius of the declination line and \( \delta \) is the Sun's declination. The table shows that the values for the Zutphen quadrant are broadly in line with those of both the slightly later Richard II quadrants and the theoretical values. Trials with different input parameters tend to support the assumed latitude and obliquity values.

### Table 1. Comparison of the normalised radii of the declination arcs on three equal-hour quadrants (assuming a quadrant of design radius 100). It is assumed that the obliquity is 24° and the latitude is 52.0°. See ref. 11 for details of the Richard II quadrants.

<table>
<thead>
<tr>
<th></th>
<th>Zutphen</th>
<th>Dorset Mus</th>
<th>Bonhams Richard II</th>
<th>Theoretical</th>
</tr>
</thead>
<tbody>
<tr>
<td>Summer solstice</td>
<td>91.2</td>
<td>89.3</td>
<td>88.9</td>
<td>88.7</td>
</tr>
<tr>
<td>Equinoxes</td>
<td>64.0</td>
<td>63.6</td>
<td>61.2</td>
<td>62.3</td>
</tr>
<tr>
<td>Winter solstice</td>
<td>24.2</td>
<td>25.2</td>
<td>23.4</td>
<td>25.0</td>
</tr>
</tbody>
</table>

A similar circle-fitting exercise can be performed on the hour lines. The results are shown in Fig. 9 where it can be seen that the circles do not display a well-defined arrangement. Measurements of the altitude angles on the equinoxes (when there is no effect from the obliquity value) indicate errors of up to 1.7° for the 3 & 9 hour arc (see Table 2). This seems to indicate that the device and/or the tables from which it was delineated were not of great precision.

### Table 2. Theoretical vs. measured angles of the Sun’s meridian altitude on the equinoxes for the Zutphen quadrant. The theoretical values use a latitude of 52° (near the optimum).

<table>
<thead>
<tr>
<th>Hour</th>
<th>Measured</th>
<th>Theoretical</th>
<th>Error (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>38.0</td>
<td>38.0</td>
<td>0</td>
</tr>
<tr>
<td>1 11</td>
<td>35.7</td>
<td>36.5</td>
<td>0.8</td>
</tr>
<tr>
<td>2 10</td>
<td>33.0</td>
<td>32.2</td>
<td>–0.8</td>
</tr>
<tr>
<td>3 9</td>
<td>27.5</td>
<td>25.8</td>
<td>–1.7</td>
</tr>
<tr>
<td>4 8</td>
<td>18.1</td>
<td>17.9</td>
<td>–0.2</td>
</tr>
<tr>
<td>5 7</td>
<td>8.5</td>
<td>9.2</td>
<td>0.7</td>
</tr>
<tr>
<td>6 6</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

**Metallurgy**

Knowing the metallurgy of an instrument provides an extra set of clues about its origin and history and also contributes to the understanding of the state of metal smelting, founding, working and distribution in medieval Europe, a topic where there is a notable absence of solid information.

The composition of the quadrant was assessed by x-ray fluorescence spectroscopy. This method has the advantages
that it is fast and is completely non-destructive and non-marking. The results are quantitative if the system is fully calibrated with appropriate specimens. The area sampled is typically a few millimetres in diameter but it is essentially a surface analysis method and hence cannot give true values for the bulk composition of the object.

The results of the analysis are shown in Table 3 which also includes four other early instruments for comparison. The first point to note is that the Zutphen quadrant is a true brass with only a small amount of tin. This is what might be expected of a ‘Continental’ instrument where there are few indigenous sources of tin. (Note that the Richard II quadrant, although ostensibly an English instrument, might well have been made in Europe, e.g. in Paris, as it was high-value courtly object. It has a high zinc content asbefits its status, i.e. it is made of high-quality brass.)

The Zutphen quadrant, and the rather later Grafendorf compendium, both have a significantly lower zinc content which is more typical of the early 14th century and might thus be regarded as consisting of rather ‘every day’ quality brass. By contrast, the two English instruments (the Canterbury quadrans novus and the Norfolk horologium) both contain a significant amount of tin (and, to a lesser extent, lead), thus tending to confirm their local source. These conclusions cannot be regarded as definitive as there are not yet enough samples in the database, but they do point to the probability of instruments, including the Zutphen quadrant, being made in the approximate locality of their use.

As well as being slightly bent, the quadrant has at least two significant cracks, starting at the 14° and 34° points on the limb and propagating inwards. These are evidence of ‘season cracking’ along grain boundaries due to the exposure in the soil and usually associated with small amounts of ammonia. Considering the long exposure, they do not suggest a particularly poor-quality brass.

### Table 3. Composition of metal alloys in wt. % as measured by XRF of the Zutphen quadrant and four other medieval time-telling instruments.

<table>
<thead>
<tr>
<th>Instrument</th>
<th>Date</th>
<th>Cu</th>
<th>Zn</th>
<th>Sn</th>
<th>Pb</th>
<th>Other</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zutphen quadrant</td>
<td>c. 1300</td>
<td>85</td>
<td>12.5</td>
<td>0.5</td>
<td>0.2</td>
<td>0.3% Ag; 0.7% Fe</td>
</tr>
<tr>
<td>Richard II quadrant</td>
<td>1396</td>
<td>78</td>
<td>22</td>
<td>Tr</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Grafendorf compendium</td>
<td>c. 1450</td>
<td>79</td>
<td>12</td>
<td>0.8</td>
<td>2</td>
<td>1.4% Fe</td>
</tr>
<tr>
<td>Canterbury quadrans novus</td>
<td>1388</td>
<td>87</td>
<td>5.3</td>
<td>3.4</td>
<td>1.5</td>
<td>0.2% Ag; 0.6% Fe</td>
</tr>
<tr>
<td>Norfolk horologium</td>
<td>??</td>
<td>77</td>
<td>12</td>
<td>7.4</td>
<td>2.4</td>
<td>0.2% Ag; 0.9% Fe</td>
</tr>
</tbody>
</table>

a. The subject of this article, measured by Dr Bertil van Os of the RCE (Netherlands Cultural Heritage Agency).
b. One of four known devices (1396–1400), this is the earliest (unsold by Bonhams in 2012); results supplied by Christopher Becker. The two devices in the British Museum have very similar compositions (results from Dr Susan La Niece and the author).
c. Excavated in Grafendorf Castle near Vienna, analysis by Prof Dipl. Ing Dr Manfred Schreiner (Akademie der Bildenden Künste), communicated by Dr Ronald Salzer.
d. Found in Canterbury by excavation, now in the British Museum. Analysis by Dr Brian Gilmour and Dr Peter Northover (Oxford University).
e. Metal detectorist’s find, analysed by Dr Brian Gilmour (Oxford University).

### Implications and Concluding Remarks

The Zutphen quadrant is an important find because, with its rather secure dating to around 1300 (it is assumed that it must be a few year older than the layer from which it was excavated) it pushes back the date of the known use of European quadrants of this type for showing equal hours by nearly a century. Its design is a simple extension of the previous, basic type of quadrans vetus for single latitudes showing unequal hours, but without the significant difficulties of using the sophisticated quadrans novus or astrolabic quadrant also appearing in Europe at the beginning of the 14th century. It seems likely that this was related, directly or indirectly, to the appearance of the first tower clocks at around this time. But it took a long time for the unequal hour system to be totally displaced and it was another century before sundials on horizontal or vertical planes and using polar-oriented gnomons were to be developed in Europe – although altitude dials (e.g. cylinder dials) for the purpose were available much earlier.

The quadrant raises a number of questions which cannot be answered yet. The first is who the user was. The likelihood is that it was a member of the educated merchant class, wealthy enough to afford such an instrument, intelligent enough to understand the method of use, and interested in knowing the time. As the various references to time-telling in Chaucer’s Canterbury Tales indicate, this was still something of a novelty. In the trading environment of a Hanseatic port, there would have been many practical reasons for needing to know the time. The quadrant would have been easily accurate enough for the purpose and its latitude of 52º would have covered a wide area of the other Hanseatic ports including, perhaps, Norwich or Lynn (later Bishop’s and then King’s Lynn) in Norfolk.

The next questions are by whom and where the quadrant was made. Other than the Parisian workshop of Jean Fusoris nearly a century later, we have no real knowledge
of mathematical instrument making in the period. The metallurgy, as well as the latitude, suggest that it was made relatively locally, or at least on the Continent. But whether there was one or several instrument-making workshops will require evidence from other sources. The case of quadrants can be usefully compared to that of astrolabes and naviculae, made in 14th-century Europe where there is a similar lack of knowledge of manufacturing centres.

ACKNOWLEDGEMENTS
I am extremely grateful to Bert Fermin, archaeologist at Zutphen, for notifying me of the find, for many useful discussions, and for permission to quote extensively from the official report (ref. 1). His colleagues Davy Kastelein, Michel Groothedde and Elisa de Vries also provided drawings and other information. Dr Jenny Cripps (Dorset County Museum) kindly arranged access to the Richard II quadrant in Dorchester and Christopher Becker provided valuable photographs and XRF data for the 1396 Richard II quadrant now in Australia. Brian Gilmour (Oxford Research Laboratory for Archaeology and the History of Art) obtained some of the XRF results. Ronald Salzer (Vienna University) provided information on the Grafendorf compendium. Michael Lowne stimulated some useful discussions.

REFERENCES and NOTES
1. For a detailed report of the excavations, with a description of the background and all the finds, see B. Fermin & D. Kastelein: Het Zutphense kwadrant – Archeologisch onderzoek in de gracht van de ringwalburg op de Houtmarkt te Zutphen, Zutphense Archeologische Publicaties 80 (July 2013).
2. See Wikipedia for an easily-accessible English description of the city of Zutphen.
3. Described fully in ref. 1.
9. Inventory no. 44178
10. See British Library manuscript catalogue.
11. Two of the quadrants are in the British Museum and a third is in the Dorset County Museum in Dorchester. In 2012 a fourth quadrant, dated 1396 and thus the earliest of the group, was discovered in Australia. It was offered for sale by Bonhams in London but failed to meet its reserve price of £150,000. For more details of these quadrants see S. Ackermann & J. Cherry, ‘Richard II, John Holland and Three Medieval Quadrants’, Annals of Science, 56 (1999), pp. 3-23. Also, C. Eagleton: ‘A King, Two Lords, and Three Quadrants’, Early Science and Medicine, 16, pp. 200–217 (2011). One of the Richard II quadrants is illustrated in F.A.B. Ward: A catalogue of

Blackamoor Cartoon
This image is part of a cartoon strip titled ‘Hobby Horses’ and dated September 1707. It must be very much contemporary with the Van Nost figures described in the previous Bulletin by Roger Bowling. Courtesy of The Trustees of the British Museum, inv. no. AN00478665

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12. The 1399 Richard II quadrant does have declination lines, for every 10° of celestial longitude (dividing each zodiac sign into three). These lines, however, were added at a much later date (possibly in the late 16th century) as shown by the different engraving hand.
13. See ref. 4.
16. The diagram in MS Ashmole 19 (f.54) is shown in Gunther (ref. X) p.172.
17. J.E. Morrison: The Astrolabe, Janus, Rehoboth Beach (2007). The formulae for the unequal hour quadrant are given on pp. 213-220 with the latitude-dependence of the deviations from true circularity being on p. 220.
19. This and the other equations may be obtained from first principles or any of the basic sundial texts.
20. ‘accurate’ here means that it was taken with the camera plane accurately parallel to the quadrant and with minimal lens distortions. This is rather difficult with the Zutphen quadrant as the plate is far from flat. As a result, CAD-fitted circular arcs are not always a very close fit to the photographs.
21. I am indebted to Dr Bertil van Os of the RCE (Netherlands Cultural Heritage Agency) for undertaking the XRF measurements and to Bert Fermin for arranging them. Other analyses are credited in the caption.