A MEDIEVAL GUNTER'S QUADRANT?

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Edmund Gunter (1581-1626) first described his eponymous quadrant as an appendix to his 1623 book *De Sectore et Radio*.[1] So a true medieval Gunter's quadrant is simply not possible. However, a fragment of an instrument excavated in Norfolk in 2009 suggests that the concept behind the instrument may possibly have been known around two centuries before Gunter developed his version.

Introduction

Many designs of quadrant have been used as forms of altitude sundials for at least a millennium. Exploring the development of the various types across the cultures of the world gives a valuable insight into the knowledge of mathematics and astronomy through history, and the types of timekeeping prevalent at any given period. In addition, the nature of the actual devices gives tangible evidence of the level of technology at the time.

The earliest quadrants tend to come from the Islamic world and were designed for unequal (temporal) hours, usually measured with 12 hours from sunrise to sunset and thus putting noon as 6 hours. At the low latitudes of the Islamic world they could be regarded as universal. This was not strictly true for Europe, where they often incorporated a moving calendar scale or cursor. These early quadrants are usually termed *quadrans vetus.*

Horary quadrants based on a similar layout but designed to show equal hours (usually counted from noon and midnight) became popular in Europe in the Middle Ages and these were sometimes combined with the older unequal hour scales on the same instrument. In addition, a very sophisticated design – the *quadrans novus* – based on the mathematical projections of an astrolabe was introduced in the late-13th century from Islamic Spain and although it never became commonplace it may have been influencial amongst astronomers and advanced thinkers.

Sixteenth and seventeenth century Europe saw a profusion of new quadrant types being developed, either because they had some practical advantage (ease of use, accuracy, universality, ability to provide large amounts of information etc.), because they were easy or cheap to make, or simply because they were fashionable and well-marketed. Amongst these, the Gunter's quadrant was perhaps the most popular with a production life of over a century. Despite being a latitude-specific device, it was optimised for simplicity and accuracy in time-telling and could also give information such as the azimuth of the sun and its place in the ecliptic. It is a hybrid design, drawing on stereography but only for the basic grid on which the hourlines are plotted. Visually, the device is readily identifiable by the pairs of inward-facing arcs which fill a broad band around the limb and meet at cusps on a common circle.

The Norfolk Quadrant

The item shown in Fig. 1 was found in March 2009 by Graeme Simmonds, a Norwich metal detectorist, in an arable field in the Norfolk Broadlands. It was towards the bottom of the topsoil, about 300 mm deep, next to an ancient trackway which continues to a ruined medieval church. There were no associated finds in the immediate vicinity though other medieval artefacts and coins have been found in the same field. Many pieces of Roman pottery and tile have also been found but there were no finds from more recent eras. The field is quite elevated (for Norfolk!) and the trackway, running roughly N-S, is not part of any major route to Norwich so the manner in which the quadrant was lost remains a mystery.

After cleaning up, the find was shown to the local expert of the Portable Antiquities Scheme[2] (now administered by the British Museum) who declared that it was part of the plate of an astrolabe and dated from the period 1500 to 1700. It was


Fig. 1. Photograph of the excavated fragment.

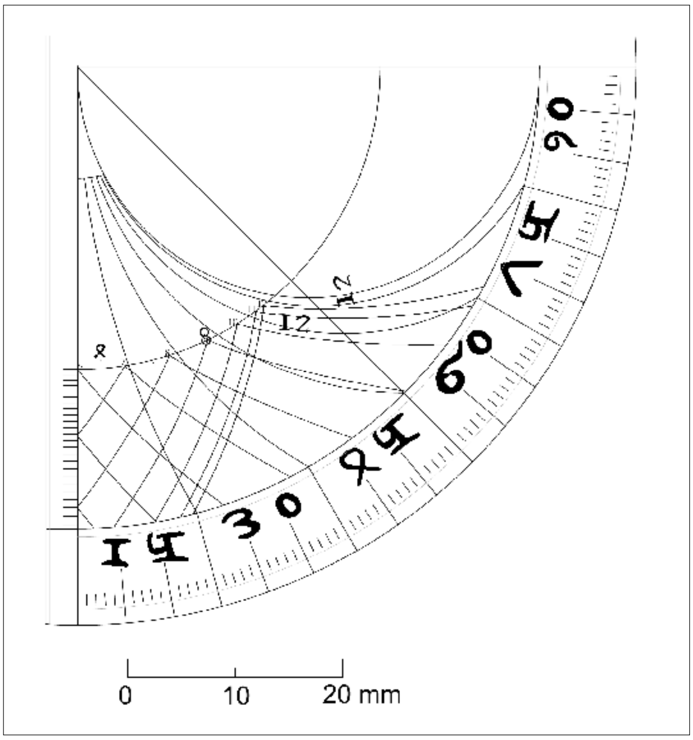

Fig. 2. Speculative drawing/reconstruction of the whole quadrant.

entered into the Scheme's database and a picture shown on their website, along with 650,000+ other items. The first person to spot the mis-identification was Glenn Grieco in the USA who correctly noticed that the numerals were medieval in style and that some of the engraved lines on the surface bore a strong resemblance to those on a Gunter's quadrant. These prompted this author to investigate further.

The find is in the form of just over half of a quadrant with an approximate radius of 52 mm (measured from the lost origin). It is clearly made of a copper-based alloy and it is about 1.0 mm thick. The left edge in Fig. 1, and the limb, are original and the right edge is where the instrument was broken, perhaps intentionally as the break is rather straighter than if, for example, it had been hit by a plough. The quadrant was rather bent and is split slightly but has now been gently flattened, though not perfectly.

The verso face is blank but the one shown has a number of engraved lines and numerals which may be divided up into three sets. The first set, around the limb, simply give a degree or protractor scale. Next inwards is a band of criss-crossing arcs which have strong similarities to Gunter's design. Finally, a small set of large-diameter arcs, passing through the origin or centre of the quadrant, represent the old *quadrans vetus* or unequal-hour quadrant. Fig. 2 gives and idealised view of what the whole instrument may have originally looked like, reconstructed mathematically from first principles.

The Degree Scale

The limb is divided into 15° segments, sub-divided to individual degrees. The segments are numbered anti-clockwise 15,30,4[5], 6[0], presumably continuing on the missing part to 75 and 90. The scale thus measures the altitude of the sun by means of a plumb-bob hanging from the quadrant origin and a pair of sights along the top radius of the instrument (all now lacking). The division into segments of 15°, rather than 10°, is unusual but not unique. The 'physician's quadrant' at Merton College Oxford, dated to 1400-1450, uses the same scheme.[3] The form of the numerals (Fig. 3), particularly the '5' and the '6' (and also the eroded '4' when viewed through a microscope), is clearly medieval, in a style that is not normally seen after the late 15th century.

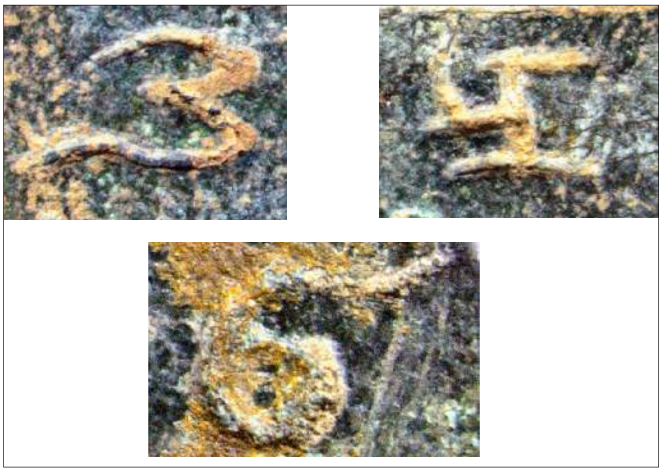

Fig. 3. Close-up of the numerals '3', '5' and '6' showing their medieval form. Photos: Dr P Northover.

Unequal Hour Lines

A set of large circular arcs, coincident at the (missing) origin of the quadrant and with their centres on the missing radius line, are the unequal hour lines found on the early form of the *quadrans vetus* (old quadrant) and also on the backs of many astrolabes. From a measurement of the sun's altitude, they give the time in the old 'seasonal' hours, dividing each day into 12, starting at sunrise. The radii and origins of the arcs can be calculated from:

$$r_n = R/2\cos(15n), \qquad x = r_n \qquad y = 0$$

where r_n is the radius of the n^{th} unequal hour before or after noon and R is the outer radius. It is also possible to find these arcs by trial-and-error with compasses, noting that the centre must lie on the meridian line (horizontal in Fig. 2) and the circumference passes through the origin and also though the point $(90-15n)$ on the altitude scale. This is probably the method a medieval maker would have used.

The arcs are independent of the design latitude meaning that the *quadrans vetus* is universal, although the underlying declination scale (rarely engraved on actual devices) is latitude dependent. The scale is calculated using the equation

$$r_\delta = R\sin(90-\phi-\delta)$$

where ϕ is the latitude and δ is the sun's declination. A very short section of an arc, barely 2½ mm long, is visible on the left hand edge quite near the apex, terminating where it reaches the cluster of unequal hour lines, represents the declination of the winter solstice for the unequal hour lines. Several of the arcs terminate on this winter solstice declination arc rather than continuing to coincide at the origin. This means that the quadrant has been designed for a particular latitude, despite its inherent universality. This feature is often found on old quadrants with equal hour arcs e.g. the 1399 Richard II device,[4] but rarely for unequal hours.

These unequal hour lines are entirely consistent with a standard medieval quadrant, the design of which can be traced back to ninth-century Baghdad.[5] Strictly, though, they should not be exactly circular but the approximation is a good one at low latitudes – where the instrument was first developed in the Islamic world – and a passably good one for medieval Europe.[6] However, the inner, semicircular, arc which represents noon appears to be labelled ‘12’ whereas for unequal hours counted from sunrise it would be expected to be ‘6’. None of the other lines are labelled.

Equal Hour Lines – Gunter’s Quadrant

The equal hour arcs which are the surprising aspect of this find are contained within a broad band between two arcs concentric with the limb. The similarity with an 18th-century Gunter’s quadrant can be seen by comparing with Fig. 4. The inner arc represents the celestial equator and the outer represents the tropics of both Capricorn (winter) and Cancer (summer). These curves result from drawing two stereographic projections of the celestial sphere onto the equatorial plane, one from the north celestial pole and the other from the south. The radial distance between the lines represents the sun’s declination (from 0 to ±23.5°) in a non-linear manner, though the scale is not sub-divided on the Norfolk device.

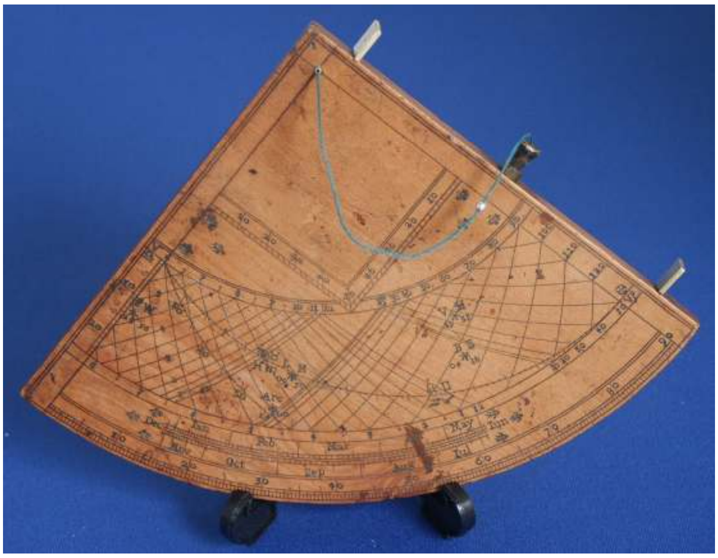

Fig. 4. A typical 18th-century Gunter’s quadrant in boxwood.

The equal hour lines occur as pairs of curves meeting on the equatorial circle and measure the time from noon (i.e. modern solar time). The curve sloping to the left from each pair is for use in the winter (i.e. from the autumnal to the vernal equinox) and that sloping to the right for the summer half of the year. The curves are not the result of a projection but come from solving the standard spherical trigonometrical equation relating the sun’s altitude, a, with the latitude, ϕ, the declination for the date, δ, and the hour angle, h:

$$\sin a = \sin\phi \sin\delta + \cos\phi \cos\delta \cos h$$

The solutions of this equation, in polar co-ordinates (δ, a) and noting that the declination scale is non-linear (with a different form to that of the unequal hour quadrant) give the lines indicated. They are invariably drawn as circular arcs which is not quite exact: a complete numerical solution shows small deviations of less than the width of an engraved line. To draw the lines without resorting to extensive numerical calculations, a medieval maker would need to consult tables to find the sun’s altitude for the equinoxes and solstices and at two intermediate declinations (one in summer and one in winter) and then fit a circular arc through the points.

In addition to the hour lines, there are ticks on the equatorial arc denoting the equinoctial half-hours for 6:30 and 7:30.

The right-hand pair of hour lines represent noon and are again labelled “12” as would be expected for equal hours. The cusp where this pair of curves meet the equatorial arc (δ = 0) gives the position where the sun’s altitude equals the co-latitude of the location for which the quadrant was designed. For the Norfolk quadrant, this has been measured to give a latitude of $\phi = 52.5° \pm 0.2°$. It is not insignificant that the latitude of Norwich is 52° 38′ N and so the quadrant appears to be locally made.

Problems

There are a number of areas of the Norfolk design which are deficient as representations of either an unequal or equal hour quadrant.

For the unequal hour portion, the labelling of the noon line is one such feature which suggests that the figure ‘12’ was a later addition made when this was the conventional time of noon – note that the use of equal and unequal hours overlapped by at least a century.[7] Another instrument which has the noon unequal hour labelled ‘12’ is the navicula-like sundial in the Whipple Museum, Cambridge.[8] This is dated to 1620.

Another problem is that some of the unequal hour lines appear to be absent. A more serious difficulty is that there is no indication of either a calendar or a declination scale. One or the other of these is needed for setting a bead on the plumb-line at a position which would allow it to indicate the time against the hour lines when the line is held taught along the current altitude line. It is, of course, possible that there was a declination or zodiac scale along the missing radius of the instrument (underneath the sights) as is seen in the drawing of Fig. 5, taken from Fale’s *Horologiographia* of 1593.[9] The instrument could be used without a declination scale by using tables to find the sun’s noon altitude for a given declination ($a_{noon} = 90° - \phi \pm \delta$) and setting the bead appropriately, though this is rather less than convenient.

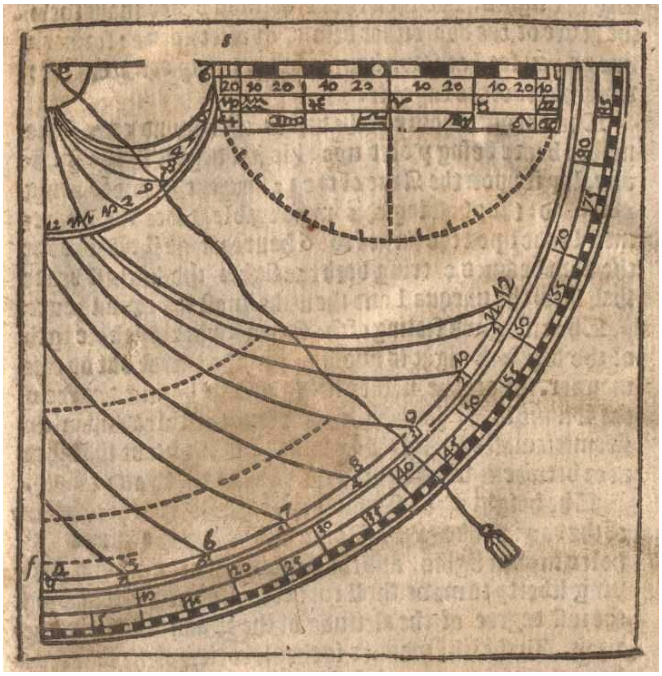

Fig. 5. A quadrant from Fale's Horologiographia showing the construction of a zodiac date scale underneath the sights.[9]

For the equal hour portion, the lack of declination or calendar scales is again crucial. Also missing from Gunter's full design, though optional for a basic time-telling instrument, are engraved arcs for the ecliptic (labelled by the zodiac signs), an arc representing the horizon, and a set of arcs giving the sun's azimuth. Another puzzle is the figure '8' which is engraved at the cusp of the lines representing 3 hours before and after noon. This looks to be a simple mistake but the absence of labelling of any of the other lines is difficult to understand.

The radii of the equator and tropics arcs of the Gunter projection are related by the same simple equation used for an astrolabe plate:

$$R_{Eq} = R_{Trop} \tan[(90-\varepsilon)/2]$$

where ε is the obliquity of the ecliptic. Note that the latitude does not appear. If an obliquity of 23.5° is assumed (although a value of 24° is not uncommon in manuscripts of this period) then the radius of the equatorial arc on the Norfolk quadrant is too large by about 2.2 mm.

Discussion

Whatever the problems and questions relating to the Norfolk quadrant, it is undoubtedly an intriguing find. It seems to represent a hybrid device and shows that the maker was interested in both equal and unequal hours and knew enough mathematical astronomy to produce an advanced instrument. It is possible that the device was only an experimental design, or that it was left unfinished for some reason.

With the potential importance of the quadrant and its relatively unknown provenance, the possibilities for its manufacture need to be considered carefully. The range of cases include:

- An outright modern 'fake'. Since the finder and the location are well attested, this option can be virtually ruled out. The lack of significant monetary value also makes it unlikely.
- A 17th- or 18th-century device for which its maker has adopted medieval numbering for some unknown reason. This seems unlikely.
- A medieval *quadrans vetus* which has later (in the 17th century or later) been brought up to date to indicate equal hours, using Gunter's scheme, by someone living in the Norfolk area. This cannot be ruled out. There is a precedent for a similar updating in the previously-mentioned 'physician's quadrant' at Merton College Oxford where the unequal hour lines have been partially erased and replaced by similar ones for equal hours.[10] In that case, though, the new lines are of the type found on quadrants from the late-medieval period, such as that made for Richard II in 1399.[11] The fact that only medieval items or earlier were found in the locality where the quadrant was excavated points to it not being a later adaption.

A study of the engraving under an optical microscope shows that the lines of the degree scale and the equal hours have a similar appearance and hence may have been engraved at the same time. The unequal hour lines, by contrast, are engraved significantly less deeply and appear to underlie the equatorial arc. Whether it was engraved minutes, years or centuries earlier cannot be judged.

If the whole instrument was indeed engraved in the medieval period, it might be wondered why such a sophisticated device should be made for and found in rural East Anglia. In fact, the city of Norwich was second in importance only to London in medieval England and its cathedral was one of the very first to have a clock: it was already being repaired by 1291. Over the period 1322-5, a new astronomical clock of considerable complexity was built at great expense (£52 9s 6d), with the clockmaker Master Roger of Stoke (who also worked with Richard of Wallingford on the celebrated St Albans astronomical clock) being involved.[12,13] Although the Norwich clock is now lost (probably by a 17th-century fire) its dial is likely to have included stereographic projections of the heavens such as those found on other astronomical clocks still extant (e.g. of the type still to be found in Prague, rather than the later and simpler type at Hampton Court Palace). Thus it is not inconceivable that the Norfolk quadrant had some connection with the Cathedral clock or its makers. Note that the projection point for the stereographic projection of an astronomical clock dial is normally the north celestial pole whereas that for an astrolabe is the south pole: both are combined in the Gunter quadrant.

If an accurate date for the instrument could be established it would be very useful in fitting it into the general time-line of quadrant development. Unfortunately, this has not so far proved possible. Initial metallographic analysis by Dr Brian Gilmour of the Materials Science-Based Archaeology Group at the University of Oxford show that it is composed of a quaternary alloy (copper plus zinc, lead and tin). They suggest, mainly on the basis of the zinc concentration, that the device is probably from the 14th or 15th centuries, just possibly from the early 16th. The alloy is assessed as being "of reasonable quality, not the metal high in arsenic, antimony, and lead which was used for domestic wares of basic quality".[14]

Optical microscopy also shows (Fig. 6) a network of very fine cracks on both sides of the device, some aligned along the engraved lines for short distances. These are stress/corrosion cracks, sometimes called 'season cracking'[15] resulting from attack by ammonia generated by decaying organic matter interacting with copper grain boundaries in the cold worked (i.e. hammered) brass plate.

Fig. 6. Optical micrograph showing a very fine stress corrosion crack which partially follows an engraved line.

P Northover

Another early quadrant design which makes use of stereographic projections is the *quadrans novus* (new quadrant) usually attributed to Prophatius (1236-1304) and sometimes described as the astrolabic quadrant as it is effectively a simplified astrolabe folded twice. Many early texts describing the design are known but only eight actual medieval instruments are extant, one of which was excavated in Canterbury in 2005 and sold at auction for a considerable sum[16-18] – it is now in the British Museum. Being based on an astrolabe but without moving parts (other than the plumb-bob), the *quadrans novus* was a very flexible and capable instrument though its many scales made it difficult to use. For basic time-telling, the Gunter's quadrant was significantly simpler which accounted for its widespread and long-lasting use in the 17th and 18th centuries.

The Gunter quadrant was not the first design to use a scale with the outer radius representing the tropics of both Capricorn and Cancer. The *Horarium Bilimbatum* (double-scaled quadrant) used straight lines to indicate the equal hours. An example can just be seen in Hans Holbein's famous painting *The Ambassadors*[19] of 1533. This design may be attributed to the first edition (1512) of Stoeffler's *Elucidatio*;[20] the version in Fig. 7 is reprinted from the 1564 edition. It is not immediately apparent that the straight lines can give an exact solution for the equal hours at all dates but the proof has been given by Alessandro Gunella.[21] Note that the non-linearity of the date scale required for this design is not the same as that for Gunter's quadrant.

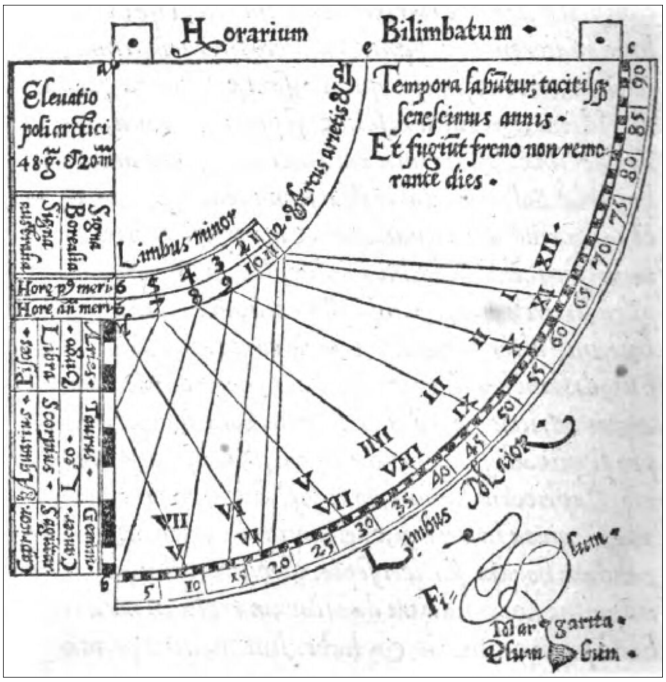

Fig. 7. The horarium bilimbatum, as shown by Stoeffler.[20]

It now seems clear that Gunter had a wealth of historical designs on which to draw when developing 'his' quadrant. He undoubtedly expanded the basic features of the underlying projections to produce a convenient and comprehensive instrument but he did not invent the basic form. Whether the 'Norfolk quadrant' was a one-off experiment or a rare survivor of a more common design which has not survived remains to be determined.

ACKNOWLEDGEMENTS

I am extremely grateful to Graeme Simmonds (Norwich Detectors Club) for allowing me to study the quadrant in detail. Peter Northover and Brian Gilmour (BegBroke Nano, Oxford Materials) kindly performed the microscopy and the metallurgical analysis and interpretation; Silke Ackermann (British Museum) offered much help and encouragement. I am also grateful for help and contributions from Glenn Grieco, Erica Darch (Portable Antiquities Scheme, Norfolk), Peter Baxandall, James E. Morrison and Mike Cowham.

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6. J.E. Morrison: *The Astrolabe*, Janus, Rehoboth Beach (2007). The formulae for the unequal hour quadrant are given on pp. 213-220 with the latitude-dependence of the deviations from true circularity being on p. 220.
7. An illustrated manuscript c. 1450 (a French translation of Henry Suso's *Horologium Sapientae*, now in the Bibliothéque Royale de Belgique, MS. Bruxelles, B.R.IV, f. 13v) shows a number of instruments including a one-handed clock with a 24-hour dial, a quadrant for unequal hours, a shepherd's dial (*chilindre*) calibrated in Italian and Babylonian hours, and portable sundials with polar-pointing gnomons.
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11. Ackermann & Cherry, *op. cit.*, (ref 4).
12. I. Atherton, E. Fernie, C. Harper-Bill & H. Smith: *Norwich Cathedral: Church, City and Diocese, 1096-1996*, Hambledon Continuum, see pp. 441-2 (1996).
13. J. North: *God's Clockmaker – Richard of Wallingford and the invention of time*, Hambledon & London (2005).
14. XRF (x-ray fluorescence) analysis of two uncleaned surface regions (back and front) were averaged to give a composition (in wt %) of: 71% Cu, 11% Zn, 8.5% Sn, 6% Pb, 1.85% Fe, 0.9% As, 0.3% Sb, 0.15% Ni, 0.1% Ag. Dr Northover comments "The effect of corrosion is rather difficult to estimate. Probably tin and lead are enhanced, arsenic and zinc are depleted, and it is a little difficult to guess for nickel and antimony. The alloy is a perfectly reasonable one for the period. I would be happy to see this alloy as being either 14th or 15th century; earlier the zinc would likely be lower and later the zinc would be higher and the tin probably lower."
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For a CV of the author, see *BSS Bull.,* 23(ii) (June 2011).

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