A UNIVERSAL ALTITUDE DIAL BY JOHN MARKE

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The King George III Collection of scientific instruments in the London Science Museum contains a wide variety of instruments and demonstration pieces from a variety of sources.[1] Some of them are very sophisticated and quite well-known, such as the 'grand orerry' attributed to Thomas Wright.[2] One item which has been largely overlooked is described as a "pocket clinometer and square in a case" and is signed "J Marke Fecit".[3] In fact, the device is a universal altitude dial of a very unusual type. It is possible that the dial was originally the property of the great chemist Robert Boyle (1627-1691). The dial is described below, together with a study of how and why it works – features which we have not seen described before and which are not immediately obvious on looking at the instrument.

Provenance

The Science Museum's King George III Collection is one of the most comprehensive surviving collections of scientific apparatus from the 18^{th} century and earlier. Its diversity is shown by two contrasting groups of apparatus. First, there is the apparatus which King George III commissioned from the instrument maker George Adams in 1761. These instruments were used by the royal family for entertainment and instruction and are expensive and elaborate. Second, there is the apparatus assembled during the 1750s by Stephen Demainbray for use in his lectures to the public. Although this apparatus was designed to demonstrate many of the same principles as those commissioned by the King, it is cheaper, simpler and more hard-wearing. The two collections came together in 1769 when Demainbray took up the post of Superintendent of the Observatory at Kew where the King's scientific instruments were housed. They were removed to King's College, London in the mid-19^{th} century and finally to the Science Museum in 1927.

The Marke dial is part of a sub-set of the King George III Collection known as the Boyle Collection. This Collection contains a number of mathematical models and books, and was incorporated into the King George III Collection in 1770. A manuscript catalogue dated 13 March 1770 lists the dial as Item No. 35, "a square brass scale plate in a case". Robert Boyle was a physicist and chemist who carried out many experiments on air, vacuum, combustion and respiration.

This collection of instruments should not be confused with the extensive set of papers owned by the Royal Society and known as the Boyle Papers.[4] These papers predominantly comprise Boyle's extant remains but, between his death in 1691 and their arrival at the Royal Society in 1769, they were the subject of serious depredations, and they also gained a certain amount of material that seems to have belonged not to Boyle but to two men through whose hands the papers passed: his executor, John Warr, and the nonconformist minister, Henry Miles, whose widow presented them to the Royal Society. Amongst the Boyle Papers are three engravings which are proofs and counterproofs of a double horizontal dial signed by John Marke and dated 1667. The relationship between these engravings, which were described in the September 2008 *Bulletin,*[5] and the altitude dial has not been established but the possibility that they both belonged to Boyle is quite good.

Fig. 1. The embossed leather case of the Marke quadrant.

John Marke

John (Johannes) Marke (c.1641- after 1673) was originally from Northampton and was apprenticed to the great Henry Sutton. He started his apprenticeship in 1655 in the Joiners' Company although he also became a Brother Clockmaker in 1667. Like his master, from whom he took over when Sutton died of the plague in 1665, he could work in brass, silver, ivory or wood and made the whole range of mathematical instruments of the time. His premises were at the Golden Ball in the Strand, near Somerset House. He made instruments for both Robert Hooke and John Flamsteed but he is probably best-known for the instruments he made for James Gregory to equip the new St Andrew's Observatory when he came to London in 1673 on a purchasing expedition. These included a new plate for the latitude of

Fig. 2(a) The clinometer side, (b) one of the sights, (c) the signature and one of the sliders.

St Andrews (56° 25') to fit to the Humphrey Cole astrolabe (already a century old) which is still in the St Andrew's Museum. The stereographic projection on that instrument is a testament to Marke's skills.

Sutton was an accomplished mathematician as well as a practical instrument maker with a reputation for delineating scales with the best accuracy of the time. He was, for example, capable of deriving the requirements for the projections on a Gunter's quadrant – then newly developed – without significant guidance and he was also responsible for major developments in the features of the double horizontal dial. It seems that Marke was a star pupil with at least some of Sutton's attributes.

Dial Description

The instrument is a combined clinometer and altitude sundial, double-sided, one side carrying the clinometer and the other the sundial for determining the time from the measured altitude of the sun. Made in brass, the dimensions are 5 × 4½ inches (129 × 115 mm) and around 2 mm thick. It still has its original gold-embossed leather case shown in Fig. 1. The instrument is capable (within limits) of operating at any latitude. When it was photographed for Ref. 1, its surface was streaked with old lacquer but this has evidently been cleaned off since, leaving it highly polished and difficult to photograph.

The description of the dial in Ref. 1 describes the scales but does not attempt to say how they should be used. The authors do, however, suggest that the scales are similar to those on a quadrant by Abraham Sharp, now at Bolling Hall.[6] A recent investigation of that instrument shows that the similarity is superficial and the functions are quite different.

The clinometer side is shown in Fig. 2. It carries the signature "*J: Marke Fecit*" but no date. The altitude scale runs 0-45[°] along the base, 45-90[°] up the right-hand side and is divided to degrees and halves. Visible near the top at the ends of the line carrying the signature are two sighting apertures (Fig. 2b) enabling the sun's altitude to be found. Not, one hopes, by actually looking through the holes but by allowing the light passing through one to fall on the other. Originally there would have been a plumb-bob on a thread suspended from a small hole opposite the 45° corner. A uniform scale at the left side runs from 0 to 10 and is divided in tenths and twentieths of the units.

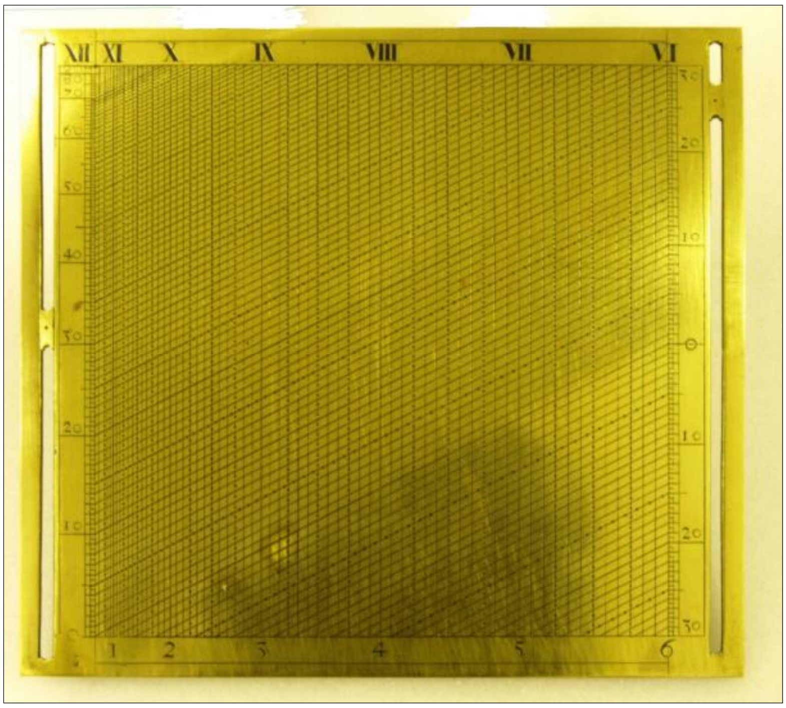

Fig. 3. The sundial time-finding side.

The altitude dial side is shown in Fig. 3. The scales are numbered but it is left to the user to know what they represent. The hour scales are along the top and bottom of the working area, morning hours in Roman numerals VI to XII at the top and afternoon hours in Arabic figures 1 to 6 at the bottom. They are spaced as the cosines of the hour-angles, the angular distance of the sun from the meridian, with the zero at the right-hand side. The upper and lower scales are joined by vertical lines at intervals of five minutes of time. The side scales are of the sines of angles, 0-90 on the left and 30-0-30 on the right, each being divided to degrees and halves. Sloping lines join corresponding points on these scales at intervals of one degree: where there are no corresponding points the lines are drawn parallel. Identifying dots are placed at the intersections of half-degrees and the hours and half-hours, and on the sloping lines at the 5-degree positions midway between the 5-minute intervals on the hour lines.

Visible from both sides are small sliders (Fig. 2c) in parallel slots which would enable a thread (now missing) to be stretched between the two sine scales on the time side. They have no function on the altitude side.

Basic Operation

The side scales are scales of the sines of altitudes. The left scale is for the sine of the meridian (noon) altitude and the right that for the altitude at 6am or 6pm (± 90º hour-angle), which in the northern hemisphere is positive for northern declinations and negative (below the horizon) for southern declinations. The zeroes denote altitude 0° on both scales and the sloping line joining them represents the horizon.

In the polar spherical triangle formula linking latitude (φ), declination (δ), hour-angle (h) and altitude (a):

$$\sin a = \sin\varphi.\sin\delta + \cos\varphi.\cos\delta.\cos h$$

On any one day and latitude, δ and φ are constant, so $\sin a$ is linear with $\cos h$ and a plot of $\sin a$ against $\cos h$ is a straight line, representing the change of altitude with hour-angle throughout the day. This is set on the dial by adjusting the sliders so that the thread is laid between the points on the side scales corresponding to the sines of the meridian altitude and of the altitude at h = 90°. As the sloping straight line between the zeroes on the side scales represents the horizon, the vertical distance between this and any other parallel line will represent the sine of a constant altitude. The point of intersection between thread and an altitude line will indicate the time, am on the upper or pm on the lower hour scale.

Detailed Operation

The first step is to derive the altitudes to set the thread on the side scales. As $\sin a$ is linear with $\cos h$ it follows that $\sin a$ at 6^h is the mean of $\sin a$ at 0^h and $\sin a$ at 12^h hour-angle. In other words $\sin a$ at 6^h is half the sum of the sines of the altitudes at upper and lower meridian transits. Calling these U and L respectively and the altitude at 6^h E:

$$U = (90-\varphi+\delta)$$

$$L = -(90-\varphi-\delta)$$

$$\sin E = ½\ \{\sin U + \sin L\}$$

$$\sin E=½\{\sin(90-\varphi+\delta)-\sin(90-\varphi-\delta)\}$$

The same result can be obtained from the polar triangle formula by putting $\cos h$ = 0:

$$\sin E = \sin\varphi.\sin\delta$$

which can be expressed:

$$\sin E = ½\{\cos(\varphi-\delta)-\cos(\varphi+\delta)\}$$

and converted to sines:

$$\sin E = ½\{\sin(90-\varphi+\delta)-(\sin(90-\varphi-\delta)\}$$, as before.

Note that L will generally be negative and E may be positive or negative depending whether δ has the same or opposite sign to φ. For southern latitudes take φ as positive and reverse the sign of δ.

In use, the instrument requires differing procedures depending on whether the declination is positive or negative:

For positive declinations:

1. Calculate U and L from the formulae above (U will be numerically greater than L).

2. With dividers measure off L on the left scale from 0.

3. Set one leg of the dividers on the value for U and the other leg lower down on the scale, thus subtracting $\sin L$ from $\sin U$ and forming the value $(\sin U-\sin L) = 2\sin E$.

4. Leaving one foot of the dividers on the value for $2\sin E$, open them to put the other foot on the zero of this scale.

5. Rotate the dividers through 90° about the zero to measure off $2\sin E$ on the horizontal base line.

6. Follow the sloping line which intersects the base at this point up to where it intersects the right-hand scale. The slope of these lines is two horizontally to one vertically, thereby dividing $2\sin E$ by two to give $\sin E$.

7. Set the dividers on the intersection point on the right scale and the zero point and mark off the same distance above centre to give $\sin E$ above the horizon. Alternatively, read the degree scale and transfer this reading to the upper part of the scale.

For negative declinations:

1. Calculate U and L as before (L now greater than U):
2. Measure U on the left-hand scale from zero with the dividers.
3. Set one leg of the dividers on the value for L and subtract the value for U to form $2\sin E$ which is negative.
4. Transfer $2\sin E$ to the lower horizontal scale as before and follow up the sloping line to intersect the vertical scale at $\sin E$. This will be correctly placed below the zero point.

In the case when δ = 0, $U = L$ and E is also 0 as may be seen from sunrise and sunset being at the east and west points of the horizon.

To set the thread to the $\cos h : \sin a$ values and find the time:

1. By adjusting the sliders, stretch the thread between the value of $\sin E$ just found and the value of $\sin U$ on the left hand scale. The height of the thread above the 0-0 horizon line then represents the value of $\sin a$ changing with $\cos h$ throughout the day. Once this has been done the setting is valid for that day, or perhaps several days at the solstices when the declination is changing only slowly.

2. Measure the sun's altitude on the clinometer side by means of the sights and plumb-bob hanging vertically.

3. On the left-hand scale, take this altitude setting and follow up the sloping line at that point. The line is parallel to the horizon and represents a constant value of $\sin a$.

4. Where the altitude line intersects the thread indicates the time, am on the upper Roman numerals or pm on the lower Arabic numerals. It is also possible to derive the times of sunrise and sunset from the point where the thread meets the horizon line.

It is noticeable that the instrument has no table of the sun's daily declination throughout the year, although there is space where the maker could have chosen to engrave one in the blank area of the clinometer side. Possibly it was provided with a printed sheet of instructions which could have included a table of declinations.

The procedure outlined above for deriving $\sin E$ is rather cumbersome and subject to error, but appears to be the only possible method using the dial itself. However, if there was an instruction sheet this could also have carried a simple nomogram for finding $\sin E$ directly from $\sin U$ and $\sin L$.

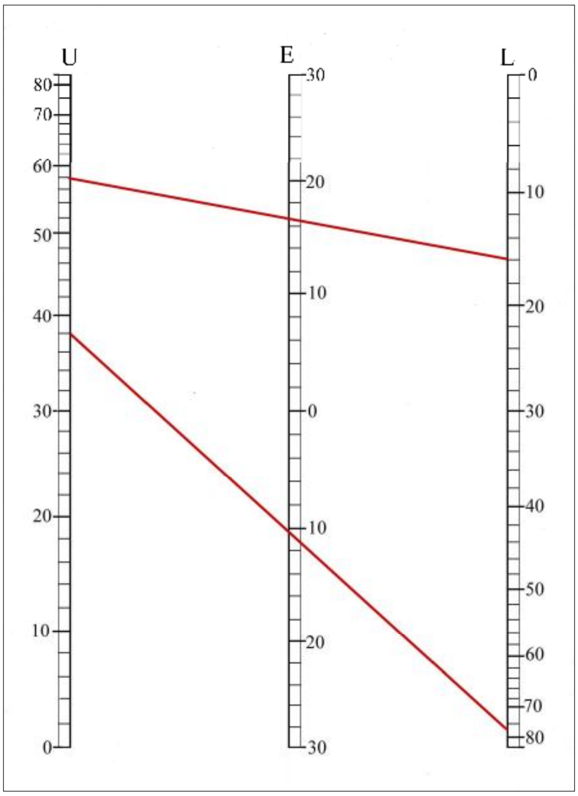

Fig. 4. A nomogram for deriving $\sin E$ from $\sin U$ and $\sin L$.

A scheme for this is shown in Fig. 4, in which a straight-edge is laid between the values of U on the left scale and L on the right. The reading of E is found where the straight-edge crosses the central scale and can be transferred to the dial scale. It will indicate directly whether E is positive or negative, to be inserted above or below the zero on the E scale. Two examples are shown on Fig. 4 with the straight-edge settings in red.

Examples of Use

An example of the use is given on the drawing of the sundial in Fig. 5 (simplified to show 2° intervals in altitude and 10 minute intervals in time). The latitude is 53° and the declination +21°. The construction lines to form $\sin E$ from $\sin U$ and $\sin L$ via $2\sin E$ and the $\cos h:\sin a$ line joining the values for $\sin U$ and $\sin E$ are shown in red. The sloping lines for three solar altitudes are shown in blue.

Two of the altitude lines (for 55º and 31º) intersect the $\cos h : \sin a$ line on the diagram enabling the time to be read directly but the third intersection for a low altitude of 13º falls off the scale to the right (as shown by dashed lines) and indicates that the time is earlier than 6am or later than 6pm. This is dealt with by measuring with dividers the distance between the intersections of the $\cos h : \sin a$ line and the altitude line on the $\sin E$ vertical and transferring this above the $\cos h : \sin a$ line. Following down the sloping line from this point to the intersection with the $\cos h : \sin a$ line will give the time, now with am on the lower time scale and pm on the upper. This construction is shown on Fig. 5. The intersection of the $\cos h : \sin a$ line with the 0-0 horizon line will show the times of sunrise and sunset, but in this case it also falls off-scale to the right: with a positive declination they too are earlier than 6am or later than 6pm. To find

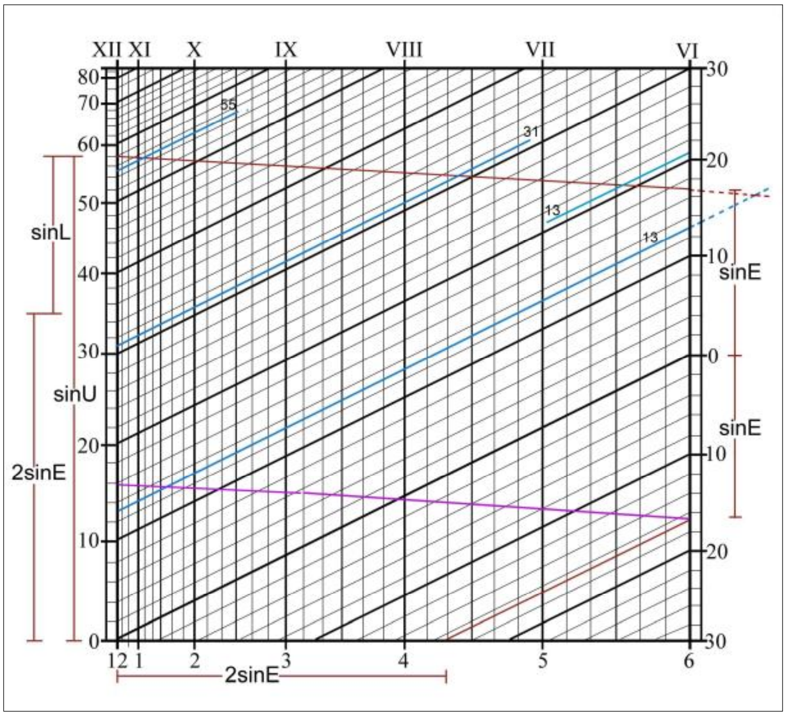

Fig. 5. Example of use for latitude 53º, declination +21º and three solar altitudes.

them, join the value of sin*E* as first found by the sloping line on the right scale to that of sin*L* on the left scale, as shown by the purple line on Fig. 5, and read the times at the intersection with the 0-0 horizon line. This could be done by laying a straight edge across the dial face to avoid disturbing the cos*h*:sin*a* line. These methods work because cos*h* is symmetrical about 6 hours, apart from the change of sign.

Latitude	+53		
Declination	+21		
U	58		
L	-16		
E	16.7		
Calc. *E*	**16.6**		
Calc. sunrise	**3:58**		
Calc. sunset	**8:02**		
Meas'd sunrise	3:56		
Meas'd. sunset	8:04		
Altitude	55	31	13
Calc. time am	**10:46**	**7:36**	**5:35**
Calc. time pm	**1:14**	**4:24**	**6:25**
Meas'd time am	10:50	7:36	5:33
Meas'd time pm	1:10	4:24	6:27

Fig. 6. Comparison of times read from Fig. 5 and calculated values.

The details of this example are shown in Fig. 6, which compares the time readings from the dial with calculated values shown in boldface type. Generally, the results from the dial are within a few minutes of the calculated values.

Another example is shown in Fig. 7 with latitude 33° and declination -19°. The comparison of the results and calculated values is given in Fig. 8. In this case for a negative declination all the dial indications fall within the 6am–6pm range and agree with the calculated values to a minute.

The two sin*U*:sin*E* lines shown on the nomogram of Fig. 4 are drawn for these two examples and confirm the values found for sin*E*.

Accuracy of the Dial

The apparent accuracy shown in these examples is in many respects unrealistic. It is one thing to make a drawing for illustration purposes using integer values for the latitude, declination and altitude but quite another to use real values with their fractions of degrees, and manipulating dividers to locate the various settings. Working in cos*h,* the time scale is very compressed and non-linear within an hour or so of noon and is correspondingly difficult to read accurately even if the thread setting is correct. Small errors in the settings will lead to inaccurate time determinations. It is considered that a quarter of a degree in the setting of the cos*h*:sin*a* thread and of the altitude reading is the best accuracy which could reliably be achieved.

The equation of the cos*h*:sin*a* line is:

$$\sin a = \sin U \cos h - \sin E \cos h + \sin E$$

This can be differentiated to find the error in *h* (*Δh*) consequent upon an error in one of the other values (*Δa, ΔU, ΔE*).

$$\Delta h = \cos a \Delta a / \sin h (\sin E - \sin U) \quad (1)$$

$$\Delta h = \cos U \cos h \Delta U / \sin h (\sin E - \sin U) \quad (2)$$

$$\Delta h = \cos E \Delta E (1 - \cos h) / \sin h (\sin E - \sin U) \quad (3)$$

For constant values of *U* and *E*:

In 1, the denominator includes sin*h* which approaches zero towards the meridian and *Δa* will increase rapidly with decreasing values of *h*.

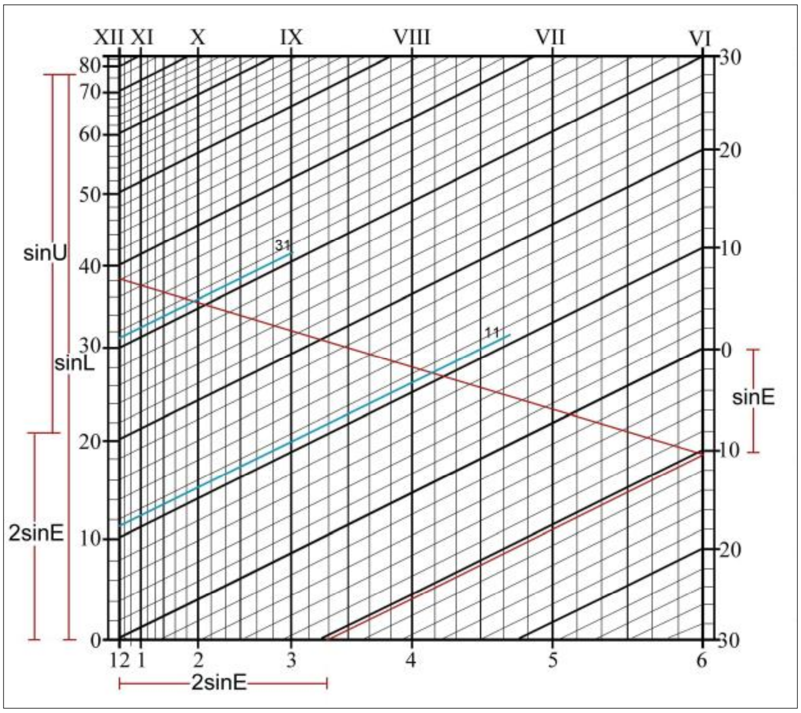

Fig. 7. Example of use for latitude 33°, declination -19° and two solar altitudes.

Latitude	+33	
Declination	-19	
U	38	
L	-76	
E	-10.4	
Calc. *E*	**-10.2**	
Calc. sunrise	**6:52**	
Calc. sunset	**5:08**	
Meas'd sunrise	6:53	
Meas'd. sunset	5:07	
Altitude	31	11
Calc. time am	**10:03**	7:51
Calc. time pm	**1:57**	4:09
Meas'd time am	10:03	7:52
Meas'd time pm	1:57	4:08

Fig. 8. Comparison of times read from Fig. 7 and calculated values.

In 2, $\cos h$ is divided by $\sin h$ which is equal to $\cot h$ and Δh will again increase rapidly with decreasing values of h.

In 3, the division of $(1\text{-}\cos h)$ by $\sin h$ is zero at the meridian and unity at h=90º. Δh will increase slowly from the meridian with increasing h.

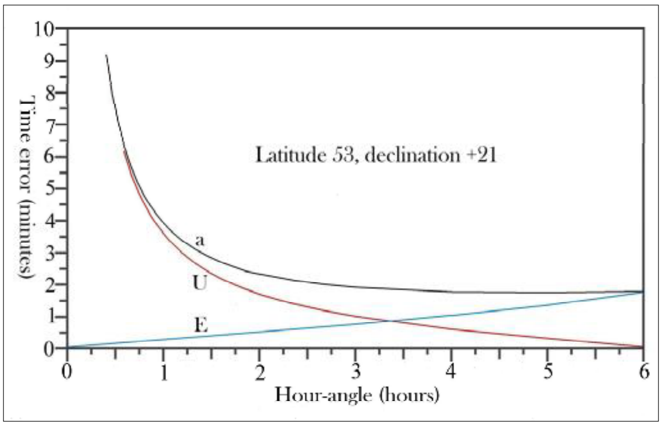

Fig. 9. Time errors resulting from small setting errors in a, U and E.

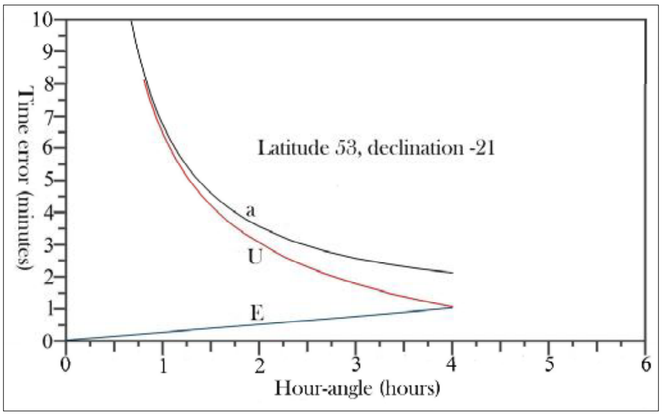

Fig. 10. Time errors from small setting errors in a, U and E.

Values of Δh derived from these expressions are multiplied by 4 to convert from degrees to minutes of time (Δt). These are plotted in Fig. 9 for latitude 53º and declination +21º as used in Fig. 5, and in Fig. 10 for the same latitude but -21º declination. Errors Δa, ΔU, ΔE are taken to be a quarter of a degree. The curve of Δt for Δa is shown in black, that for ΔU in red and that for ΔE in blue. They confirm the conclusions noted above and comparison of the two figures will show that the errors are larger at lower declinations.

In practical use there are likely to be errors in the position of more than one of the lines and the effects will combine in the incorrect time readings. The signs of the errors may be positive or negative and the line errors could add together or partially cancel out. For example, if *a* and *U* were approximately equal and of opposite sign, the accuracy of the time readings near the meridian would be enhanced.

The time errors also depend on the latitude and are larger at higher latitudes. This arises because, as can be seen by comparing Fig. 5 with Fig. 7, at the higher latitude the $\cos h : \sin a$ line makes a more acute angle with the altitude lines and a small error in one or the other will have a proportionately larger effect on the time reading. A lower declination has a similar effect on the $\cos h : \sin a$ line.

Use at Low and High Latitudes

Although the instrument is nominally usable at all latitudes, there will be complications. Within the tropics, the calculation for *U* can give a result greater than 90º: this implies that *U* and *L* are both on the same side of the zenith and is corrected by subtracting *U* from 180º. Within the Arctic Circle (use within the Antarctic Circle is perhaps unlikely!) the sun may be above the horizon for 24 hours and *L* will be positive. In this case subtract 90º from the calculated value for *L* and add the result to the value for *U* on the left-hand scale instead of subtracting. However, the derived times at very high latitudes will become most unreliable if small errors are present. In addition, the altitude becomes more nearly constant throughout the day and at the pole (to take the extreme case) does not change at all.

Discussion and Conclusions

This appears to be the only known example of this type of instrument, although it may be thought that having made one Marke would perhaps have made others. Most altitude dials are made for fixed latitudes and this universal device would have been useful for travellers in an age of rapid development of commerce and exploration.

The dial can be regarded as a nomogram for solving the spherical triangle for the special case with the hour angle equal to 0° and 90°. It is thus a simplified version of the more general 'geometrical square' described by the London-based mathematician Samuel Foster in 1659.[7] Foster's works were well-known to Henry Sutton and probably Marke too. At present, the inventor of the Marke form is open to speculation. Other uses of nomograms in gnomonics have been discussed by Sawyer.[8]

The lack of a sun's declination scale on the instrument remains a puzzle. This data can be read off directly from a double horizontal dial, or even a print of such a dial. Thus the existence of such a print, also signed by Marke and associated with Robert Boyle,[5] raises an intriguing possibility that the two instruments were in some way linked, perhaps even being commissioned as a pair. However, the sizes of the instruments (that is, the scale length of the altitude dial and the radius of the horizon circle of the double horizontal dial) are not the same and so some possible combined operations, using dividers to transfer values from one instrument to the other, are precluded, thus lessening the likelihood of their being commissioned together. Nevertheless, the connection between Marke and Boyle remains an interesting area for further investigation.

ACKNOWLEDGEMENTS

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