LINES OF DECLINATION AND TWO SEVENTEENTH CENTURY DIALS

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INTRODUCTION

Of the many lines of 'furniture' to be found on sundials, probably the most common ones are 'lines of declination'. As the sun moves north or south throughout the year its position in declination can be indicated by the shadow of a small index (called the nodus) falling on the appropriate line. These lines can be identified either by declination or by date since declination correlates with the calendar. In the former case, a simple arrangement is to indicate the position when the sun enters a zodiacal sign: often these are reduced to only the solstices (signs of Cancer and Capricorn) and the equinoxes (Aries when the sun is northbound, Libra when southbound). At present, the sun's declinations at entry to each sign are Cancer +23.44°, Gemini and Leo +20.15°, Taurus and Virgo +11.47°, Aries and Libra 0°, Scorpio and Pisces -11.47°, Sagittarius and Aquarius -20.15° and Capricorn -23.44°. Dates can be chosen to represent occasions such as anniversaries: birthdays, weddings….. However, such date lines are not unique to one date, the nodus shadow will fall on a line twice in the course of a year (except at the solstices). For example, lines for both May 8 and August 5 will catch the shadow when the sun's declination is +16º. Declination lines are uncommon on horizontal dials, although present on several very early examples.

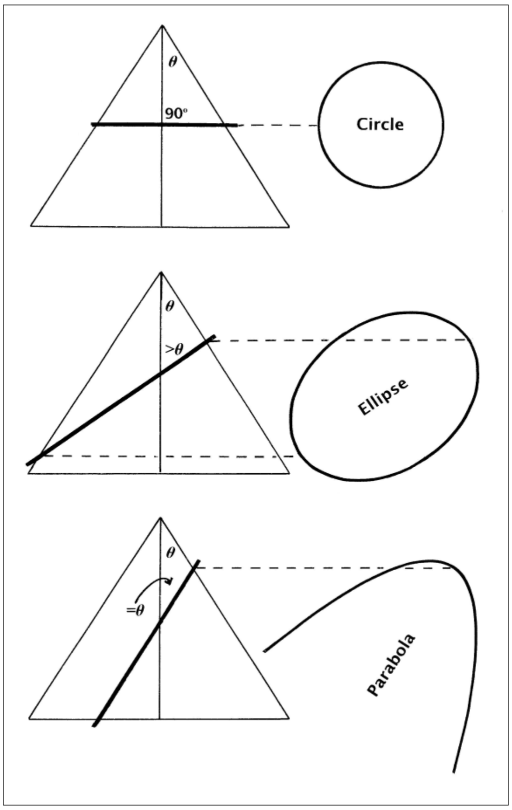

Fig. 1. Conic sections: circle, ellipse and parabola.

CONIC SECTIONS

It is well known that a declination line is a section of a cone called a hyperbola which is represented by the edges of the cone revealed by the section. As shown in Fig. 1, other sections are possible. If the cone is cut perpendicular to the axis a circle is produced. A cut which meets the axis at an angle greater than the cone semi-angle (θ) gives an ellipse, and the nearer the approach of the section to θ the more elongated the ellipse will be. A parabola is given by a cut which meets the axis at angle θ. The circle and ellipse are closed curves since the section cuts across the cone: the parabola is an open curve; the section does not meet the opposite edge.

Fig. 2 shows the case of the hyperbola. The cut is taken meeting the axis at an angle less than θ. Suppose now that an identical cone is placed vertex to vertex and on the same axis: the section will also cut this second cone as shown, producing two branches. Perhaps rather surprisingly, the two hyperbolae produced are duplicates, the one being the reflection of the other in a line midway between them and perpendicular to their centre-lines. In this way hyperbolae have four-fold symmetry and again are open curves. The

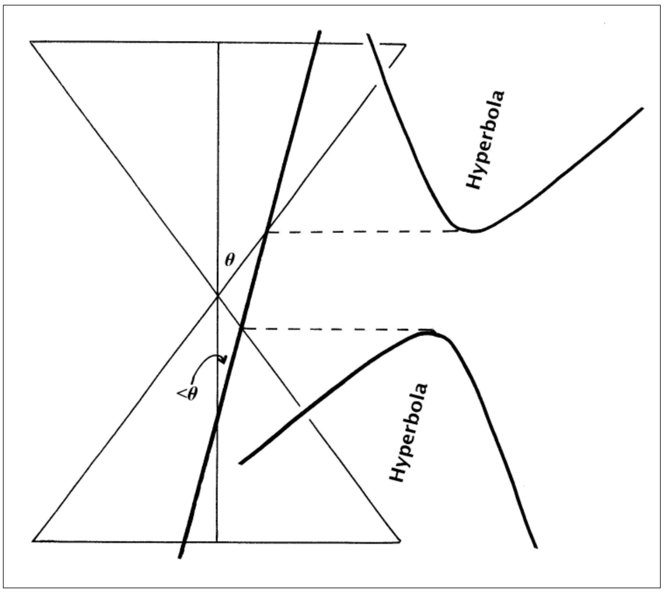

Fig. 2. Conic sections: the two branches of a hyperbola.

parabola is a unique curve and represents the boundary between the infinity of ellipses on one side and of hyperbolae on the other.

The Application Of Conic Sections To Sundials

This article concentrates mainly on horizontal dials. Leaving aside for the moment the cases of circles, ellipses and the parabola, first consider how hyperbolae arise on such dials. Fig. 3 shows a section of a horizontal dial along the meridian and two declinations of the sun throwing shadows of the nodus on the dial plate. As the sun moves across the sky during the course of a day the shadow will trace out a part-cone which is intercepted by the dial. The axis of the cone is the style and its semi-angle θ is (90º-δ) where δ (the sun's declination) is taken unsigned. The angle between the style and the dial plate is of course the latitude φ and if φ is less than θ the arc of the intersection of the cone and the plate will be a hyperbola. If, as shown in Fig. 3, the two declinations are equal but of opposite sign the values of θ of the two shadow cones will be identical. As they meet vertex to vertex at the nodus and have a common axis in the style the conditions of Fig. 2 will be met and the hyperbolae produced will be mirror images. Although it is of course well known that declination curves are hyperbolae, it is perhaps not so well known that arcs of equal but opposite declination duplicate each other in this way. This may not be obvious on an actual dial as the diverging hour lines will tend to disguise it. In Fig. 3 the line passing through the nodus at a right angle to the style is the shadow at the equinoxes which projects onto the dial as a straight line perpendicular to the meridian. The declination arcs are symmetrical about a perpendicular through the mid-point between them, but this is not the equinoctial line.

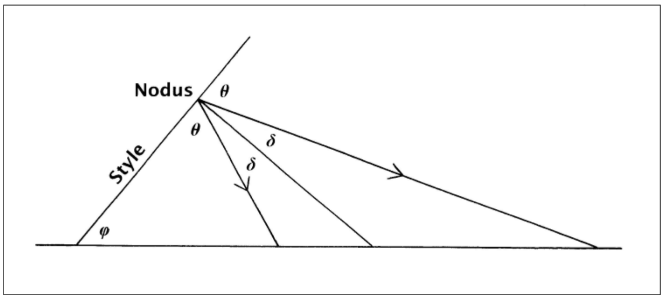

Fig. 3. The meridian shadows cast by the nodus.

Declination lines will appear as other conic sections under certain conditions. For an equatorial dial, the section of the cones of declination by the dial plane is orthogonal to the cone axis and the lines are arcs of circles. For dials at high latitudes, if (90-δ) is equal to the latitude the line is a parabola, and if less than the latitude it will be an ellipse or part of an ellipse.

There can be a choice in the type of the nodus: generally it takes the from of a small nick in the style but is sometimes made as a short cross-piece. The nodus height above the dial plane is also a matter of choice: the greater the height the wider the separation of the declination lines will be but the coverage in time will be lessened.

Delineating The Declination Lines

Methods of inserting lines of declination can be graphical, mechanical or by calculation. Graphical methods are given by Mayall[1] and by Waugh[2] (credited to William Leybourn[3]). A difficulty with such methods is that the construction lines can meet at an acute angle with the likelihood of errors unless very carefully drawn. Trigons are (or were) extensively used: in its simplest form a trigon is a mechanical device with an axis which replaces the gnomon and can be set to the appropriate latitude and declination to sweep out the lines on the dial face.[4-7]

Various methods of calculating the lines have been proposed and in these days of computers (or even pocket calculators) calculation must surely be the preferred method.[2,8-10] Some methods are based on conversion of altitude and azimuth to rectangular coordinates on the dial plane, but the coordinates can be calculated directly from latitude, declination and hour-angle. Here we consider the gnomonic projection on which plane dials with a polar gnomon are based. The nodus is taken to be at the centre of the hemisphere of the sky and positions are projected through the nodus on to the dial plane. The nodus lies within and at the centre of circles which divide the sphere equally (great circles) and project as straight lines. All other circles whose planes do not pass through the nodus (small circles) project as conic sections. In the case of a horizontal dial, azimuths of the sun are reproduced as angles from the sub-nodus relative to the sub-style and are at distances from the sub-nodus proportional to the cotangent of the altitude.

Several formulae for the gnomonic projection on a horizontal dial are possible. One of the most straightforward is:

$$x = n\{\cos P.\tan h/\cos(P-\varphi)\}, \quad y = n\tan(P-\varphi)$$

where

$$P = \tan^{-1}(\tan\delta/\cos h)$$

n is the vertical distance of the nodus above the dial plane, φ is the latitude, δ the declination, and h the hour-angle. The expression for P will fail when h=90º ($\cos h$ = 0). In this case take:

$$x = n/(\sin\varphi\tan\delta), \quad y = n/\tan\varphi$$

The coordinates x and y are measured from the sub-nodus as origin, x orthogonal to the noon line and y in the noon line. The signs of y are such that positive values refer to directions to the north of the sub-nodus and negative values to the south. The gnomonic projection reverses these directions: positive y is plotted towards the south of the dial-plate and negative y to the north. Positions in x are symmetrical either side of the meridian and need only be calculated for one side. Points found for integral hours should lie on the appropriate pre-drawn hour lines, but it would be advantageous to calculate positions at closer intervals to facilitate drawing the curves.

The formulae can be adapted to dials in orientations other than horizontal. We note that a vertical direct south dial is effectively a horizontal dial for a latitude (φ') 90º away from the desired location (for example -38º for a dial in +52º latitude) and by putting this effective latitude in the formulae the declination lines can be derived. For declining dials the standard dial formulae will give the required values: the gnomon angle (usually called the style height) is φ' and just as with the normal hour lines the hour positions are taken as (h-DL) where DL is the difference of longitude. Then in the formulae use φ' and (h-DL). The resulting x,y coordinates are referred to an origin at the sub-nodus. However, the axes are aligned not to the dial's noon line but to the sub-style line (which is the noon line at the longitude of DL).

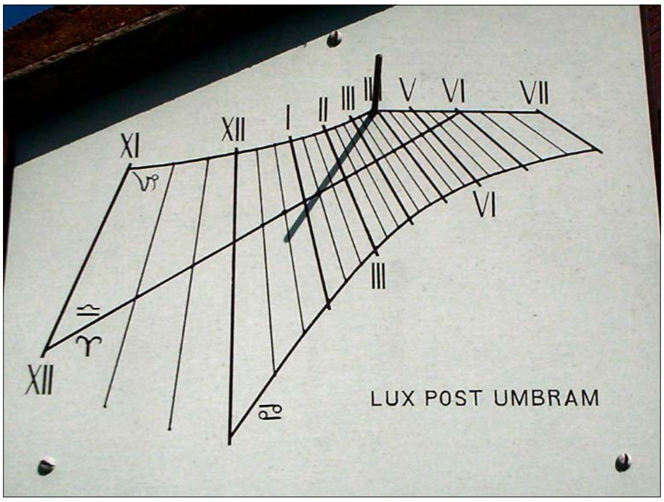

Fig. 4. A pin gnomon dial with declination lines.

Dials which consist of a pin gnomon with its apex as the nodus can be delineated by the gnomonic projection formulae without separate calculation of the time lines. The vertical declining dial shown in Fig. 4 was drawn in this way. The latitude is 50·85º and the declining angle is 52º west of the meridian: the points for the solstices and equinoctial line were calculated and plotted, then joined with the curves for the solsticial lines and straight lines for the equinoctial and time lines. The horizon is represented by the horizontal straight line which passes through the sub-nodus and the intersection of the 6pm and equinoctial lines.

TWO SEVENTEENTH CENTURY DIALS BY ISAAC SYMMES

Declination lines have appeared on English horizontal dials since the very earliest days of dialmaking, although they rather went out of favour on London-made dials after about 1630. Two interesting examples, c.1600, are by the London


Fig. 5. Dial plate of the Symmes dial at the Science Museum. The right hand side of the plate is covered with a transparent overlay showing the correct delineation.

clockmaker Isaac Symmes. (See the appendix for biographical details of Symmes.) The two sundials are in museum collections, one dated 1609 in the Science Museum in London and the other in the Oxford Museum of the History of Science. The two dials are different sizes: the Science Museum dial is 310 mm square with cut-off corners, the Oxford dial is 180 mm square. The Science Museum example is in rather an eroded condition: a photograph of the dial plate is given in Fig. 5, shown with a transparent overlay to assist in the interpretation. The Oxford dial is better preserved and is shown in Fig. 6. The dial plate carries the maker's signature: "Isaack Symmes Gouldsmith and Clockmaker". (The date of 1755 and the initials R + I are

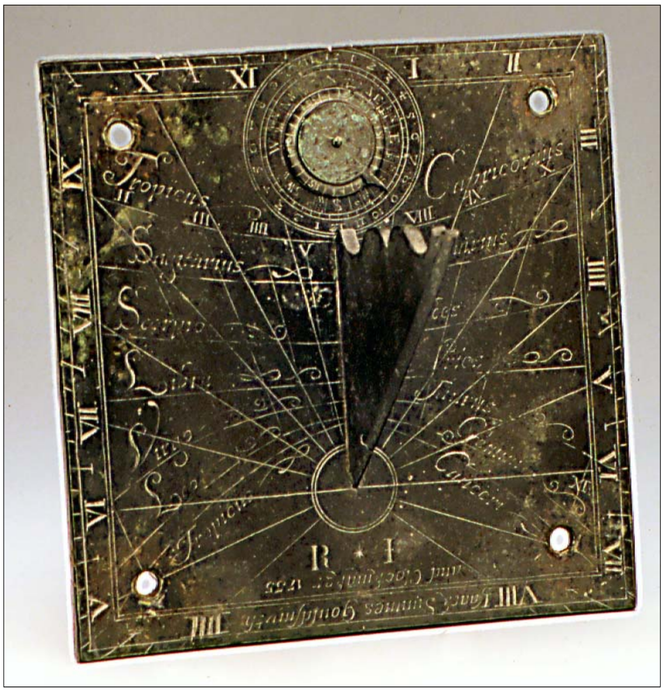

Fig. 6. The Symmes dial at Museum of the History of Science, Oxford .

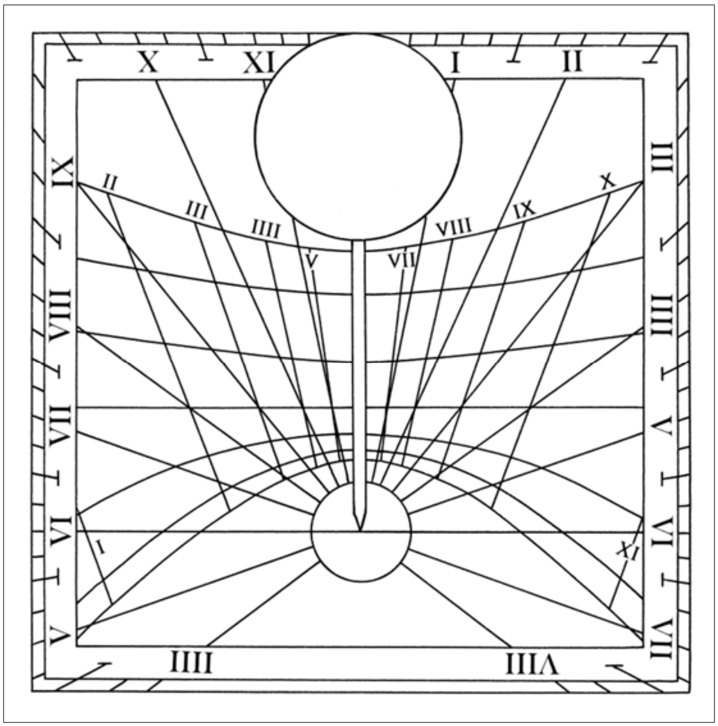

Fig. 7. Drawing of the dial plate of the Oxford dial.

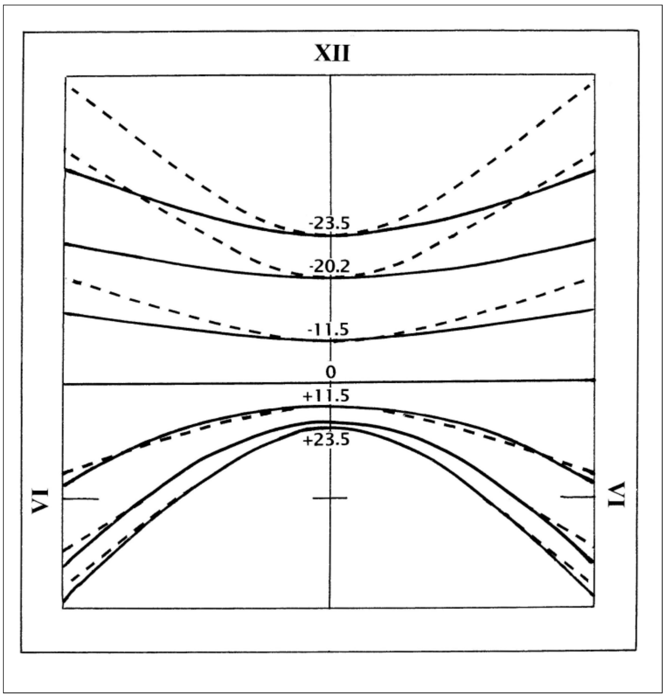

Fig. 8. The incorrect positions of the declination lines on the Oxford dial. Dotted lines show correct shapes, solid lines are the lines on the dial.

apparently later additions.) Lines of declination are given for the entry of the sun into each zodiacal sign, identified in elaborate script. There are also'seasonal hours' and a lunar volvelle for time-telling by the moon. Detailed measurement of the angles of the hour lines shows that the Oxford dial is made for a latitude of 52·5º±0·6º. It is also possible to analyse the latitude for which the declination arcs are drawn by measurement of their intersections with the meridian line: again the best fit is obtained at latitude 52·5º with line error ±0·4mm.

For additional clarity, a drawing of the dial plate is shown in Fig. 7 with the lettering and volvelle omitted, but showing the seasonal hours and the declination lines. More about the seasonal hours and the volvelle later, but first look at the declination lines. It is obvious that they do not conform to the necessary conditions outlined above: the arcs for zodiacal signs south of the equinoctial line (Scorpio -Pisces, Sagittarius-Aquarius and Capricornus) are much less curved than the corresponding ones to the north (Virgo-Taurus, Leo-Gemini and Cancer).

Fig. 8 is a drawing of the dial plate showing just the declination lines of the Oxford dial. The arcs as they appear on the dial are solid lines and their correct positions are broken lines. The lines for northern declinations are reasonably in agreement with their correct places, being if anything rather too strongly curved. Although the southern ones are correct on the noon line they depart from their true positions at hour angles away from the meridian. In fact the southern lines are of approximately the correct shape for declinations less than the true ones and are shifted bodily along the noon line so that their meridian positions are correct. Without knowing how the lines were drawn it is not possible to say with any certainty how the error has occurred. It is unlikely that the lines were derived from calculations: at this date they were probably drawn either by some graphical method, from tables, or by the use of a trigon.

The Science Museum dial has similar errors in the declination lines and there is another anomaly in that the latitude for which the hour lines are drawn does not agree with the meridional positions of the declination lines. The hour lines are for latitude 50·0º±0·2º but the best fit for the latitude of the declination lines is 51·5º with line error ±0·2 mm. The gnomon angle is 50·8º ±0·2º, neatly between the two.

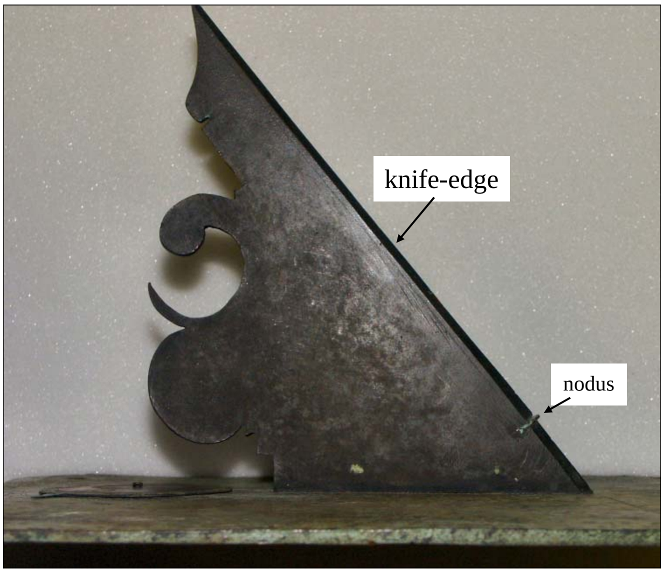

Fig. 9. The gnomon of the Science Museum dial. The chamfered style and the nodus are indicated.

Neither of the dials has a noon gap to accommodate the width of the gnomon. Instead, the style is chamfered to a central edge which will lead to inaccuracies in the time readings near noon, as the shadows will be cast by the shoulders of the chamfer, not the centre. This is not important in the case of the Oxford dial, the hour lines in this area are covered by the lunar volvelle.

The ornate gnomon of the Science Museum dial is shown in Fig. 9. The nodus and the chamfered style are depicted.

The Seasonal Hour Lines

Running between the declination lines for the solstices on the two dials are lines identified with Roman numerals I-V and VII-XI (the meridian line is VI). These are seasonal hours which divide the sunrise to sunset interval into 12 equal parts. The lengths of the 'hours' on any particular day thus depend on the length of time the sun is above the horizon. Since in the latitude of these two dials the sun is up for about 8 hours in midwinter and about 16 hours at midsummer, the length of a seasonal hour can vary from about 40 minutes (of ordinary time) to about 80 minutes.

To delineate the seasonal hours it is necessary only to derive their hour points on the declination lines for the solstices and join them with a straight line. As a check, the line should pass through the intersections of the dial hour lines with the equatorial line: at the time of the equinoxes the sun is up for twelve hours and a seasonal hour is the same length as an equal hour of solar time. The seasonal hours for intermediate declinations do not quite lie on a straight line[11] but their departure from straightness is only a minute or two and is not significant on the scale of the dial face. The method involves first finding the actual hour-angles of the sun at the seasonal hours. These are then plotted at the appropriate points on the dial time calibrations and on the declination lines. The hour-angles could be found by taking the sunrise-sunset interval (the diurnal arc of the sun) from the times given in almanacs divided into twelfths.

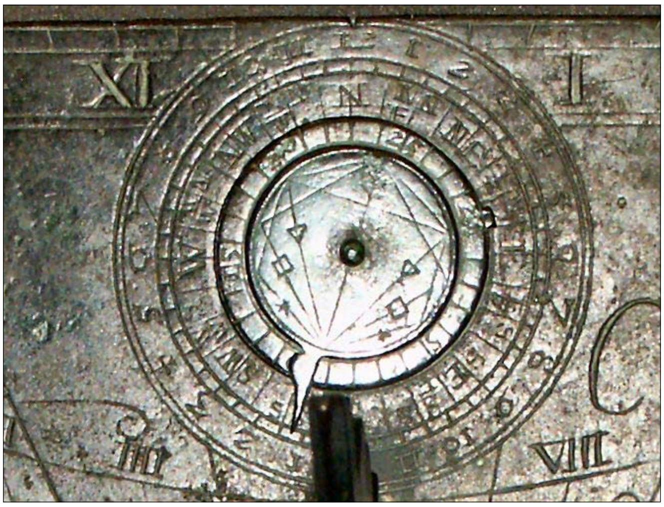

Fig. 10. The lunar volvelle of the Oxford dial.

Since the gnomonic projection does not show the horizon, the times cannot be found from the dial but could be found from a stereographic projection, which includes the horizon. Rohr[6] gives the hour-angle of the sun at sunrise for the solstices over a range of latitudes. These values are also the semi-diurnal arcs, the intervals between sunrise or sunset and the sun's meridian passage. For any declination, the semi-diurnal arc can be derived from

$$\cos^{-1}(\sin\delta/\cos\varphi)$$

and then divided into sixths. Then these points can be plotted as before, or the gnomonic formulae given earlier can be used to derive their x, y coordinates.

The Lunar Volvelle

A picture of the lunar volvelle on the Oxford dial is shown in Fig. 10. The operation of this uses the compass bearing of the moon combined with the age of the moon to tell the solar time. From the outer ring inwards, the first shows hours in the 2×12 hours system, the next has a compass rose in 32 points (only alternate ones are labelled). The inner rings rotate independently, the outer one of the two carries the age of the moon from new, 1 to 29 days labelled at 5-day intervals. The space between 29 and day 1 is wider than others to allow for a 29½ day lunation. A small projection next to 29 is an index for setting the compass direction of the moon. The inner ring carries a large index with a straight edge and has symbols indicating some phases of the moon from which the age can be determined approximately if this is unknown. New Moon and Full Moon are adjacent to and opposite the pointer. First Quarter and Last Quarter are shown by the lines through the small squares where these meet the edge of the ring. The other lines with small crossed lines or triangles are the occasions when the moon appears to be one-quarter and three-quarters illuminated.

To use the volvelle, the straight edge of the larger index is set to the moon's age (the setting is valid for any one night) and then both circles are rotated together to set the smaller index to the compass direction of the moon. The time is then read on the outer circle from the point of the larger index.

Apart from the intrinsic inaccuracies in finding time from the moon,[12] this instrument will introduce further errors. The time is found from the azimuth of the moon measured in the plane of the horizon, but should be derived from the hour-angle, in the plane of the equator. With the moon at a high declination and in the eastern or western sky, the additional error could be an hour or more.

The volvelle on the Science Museum is badly eroded and the inner circle is missing, but it evidently operated in the same way, with the addition of a pictorial means of depicting the phase appearance of the moon at any age. This could be used to derive an unknown age or (perhaps more likely) to show the appearance of the moon at any age as a guide to its brightness, to assist night-time travellers.

A METHOD OF DRAWING DECLINATION LINES

Should any diallist wish to draw the declination lines, the method shown in Fig. 11 may be found preferable to the Leybourn method. It is based on the geometric properties of the double hyperbola using the major axis, the linear eccentricity and the focal points.

1. On a horizontal line MMʹ which represents the dial substyle, draw a perpendicular equal to the nodus height *n*. N is the nodus.
2. Draw the style through N at the latitude angle φ.
3. Draw NE perpendicular to the style and draw a line from E perpendicular to MMʹ. This is the dial declination line for the equator at δ=0º.
4. From N draw two equal but opposite declination angles $+\delta$ and $-\delta$ to meet MMʹ in A and B. AB is the major axis.
5. Find the mid-point of AB at O.
6. Draw a line at the latitude angle from O to meet NB at D.
7. Measure the distance ND. (Alternatively, measure NA and NB and take the mean length.) The quantity so found is the linear eccentricity *e.*
8. On MMʹ measure off the distance *e* on either side of O to give points F_1 and F_2. These are the foci of the hyperbolae.
9. From F_1 draw arcs of circles at arbitrary radii r_1, r_2, r_3……
10. From F_2 draw arcs of radii r_1+AB, r_2+AB, r_3+AB…. to intersect the corresponding arcs from F_1. For clarity, only four intersections on each arc (more would be preferable) on one side of MMʹ are shown on Fig. 11. Those on the other side of MMʹ can be drawn at the same time.
11. Joint the points so found with a smooth curve passing through A. This is one branch of the hyperbola. From the properties, at any point P on the curve the distance from the further focus is equal to that from the nearer focus plus the major axis: $PF_2=PF_1+AB$.
12. Repeat the construction for the other branch of the hyperbola through B: $PF_1=PF_2+AB$.
13. Repeat from step 4 for other declination pairs as required.

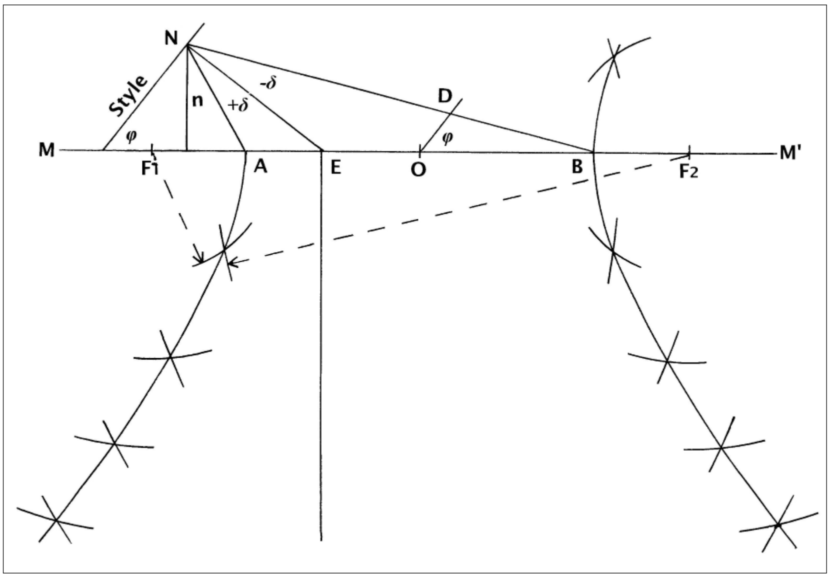

Fig. 11. Delineating the declination lines.

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APPENDIX – ISAAC SYMMES

Isaac Symmes (born c. 1580, d. November 1622) was a London clockmaker.[13-17] His name is sometimes written Isaack Simmes or countless other variants, possibly because he is said to have been of French descent[15,17] although his father is described on his apprenticeship indenture as "Roger Symes clarke of London". He was apprenticed to John Humphrey of the Goldsmiths' Company in 1596, later being turned over to Richard Lytler and being made free in January 1604. He married Emma Howe in March the same year. They lived first in the area of St Botolph, Aldgate, and, after 1612, at Houndsditch just outside the City. Judging by his will,[18] he became quite prosperous and was a well-regarded member of the community, leaving a number of gifts to the workers and poor of the district.

Just before his death he was one of the signatories to an appeal to King James I for the formation of a separate clockmakers' guild in an effort to keep out 'foreign' workers: the appeal was unsuccessful at the time with the Clockmakers' Company not being granted its charter until 1631.

Symmes is best known as a watch maker though his will indicates that he also made clocks. A particularly fine verge watch with alarm has been described by Thompson.[14] It shows that Symmes was inventive as well as a good craftsman. The two dials with declination lines and moon volvelles studied in this paper are described by Turner[13] who also shows a simpler dial dated 1610 in a private collection. In addition, Loomes[17] found another simple dial, unfortunately with an inappropriate replacement gnomon, which is now in the Clockmakers' Museum, Guildhall. A fifth dial, dated 1614, is said to be at Ridlington, Rutland.[19] Five watches signed by Symmes are known. Some exhibit very fine engraving and gilt-brass plates, as might be expected from someone who trained as a goldsmith; indeed, Symmes describes himself on his Science Museum and Oxford dials as "Gouldsmyth & Clockmaker at London".

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